From NewHope to Kyber

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"In the past, people have said, maybe it's 50 years away, it's a dream, maybe it'll happen sometime. I used to think it was 50. Now I'm thinking like it's 15 or a little more. It's within reach. It's within our lifetime. It's going to happen."

—Mark Ketchen (IBM), Feb. 2012, about quantum computers

The end of crypto as we know it

Shor's algorithm (1994)

- Factor integers in polynomial time
- Compute discrete logarithms in polynomial time
- Complete break of RSA, ElGamal, DSA, Diffie-Hellman
- Complete break of elliptic-curve variants (ECSDA, ECDH, ...)

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- Consequence: **Want post-quantum PFS crypto today**

Ring-Learning-with-errors (RLWE)

- Let $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Let χ be an *error distribution* on $\mathcal{R}_q$
- Let $\mathbf{s} \in \mathcal{R}_q$ be secret
- Attacker is given pairs $(\mathbf{a}, \mathbf{as} + \mathbf{e})$ with
  - $\mathbf{a}$ uniformly random from $\mathcal{R}_q$
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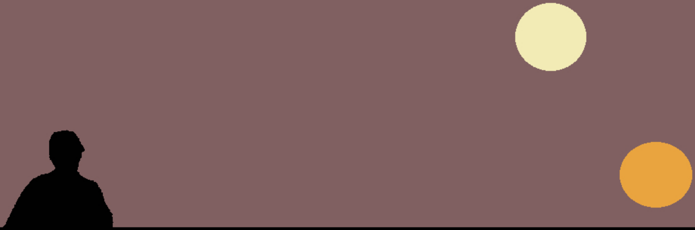
- Common choice for χ: discrete Gaussian
- Common optimization for protocols: fix $\mathbf{a}$

Alice (server)		Bob (client)
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{\mathbf{b}}$	$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
	$\xleftarrow{\mathbf{u}}$	

$$
\begin{array}{llll}
\text{Alice has} & \mathbf{t} & = \mathbf{u}\mathbf{s} & = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}'\mathbf{s} \\
\text{Bob has} & \mathbf{t}' & = \mathbf{b}\mathbf{s}' & = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}\mathbf{s}'
\end{array}
$$

- Secret and noise polynomials $\mathbf{s}, \mathbf{s}', \mathbf{e}, \mathbf{e}'$ are small
- $\mathbf{t}$ and $\mathbf{t}'$ are *approximately* the same

POST-QUANTUM KEY EXCHANGE



A NEW HOPE

ERDEM ALKIM

LÉO DUCAS

THOMAS PÖPPELMANN

PETER SCHWABE

USENIX Security 2016: NewHope

- Improve IEEE S&P 2015 results by Bos, Costello, Naehrig, Stebila (BCNS)
- Use reconcilation to go from approximate agreement to agreement
  - Originally proposed by Ding (2012)
  - Improvements by Peikert (2014)
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- Very conservative parameters ($n = 1024, q = 12289$)
- Centered binomial noise ψ_k (HW(a)$-$HW(b) for k-bit a, b)
- Achieve ≈ 256 bits of post-quantum security according to very conservative analysis
- Higher security, shorter messages, and $> 10\times$ speedup

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- Choose a fresh parameter **a** for every protocol run

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NewHope in the real world

- July 7, 2016, Google announces 2-year post-quantum experiment
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- November 28, 2016: *"At this point the experiment is concluded."*

Conclusions for Google's experiment

"[. . .] we did not find any unexpected impediment to deploying something like NewHope. There were no reported problems caused by enabling it."

Conclusions for Google's experiment

"[. . .] if the need arose, it would be practical to quickly deploy NewHope in TLS 1.2. (TLS 1.3 makes things a little more complex and we did not test with CECPQ1 with it.)"

Conclusions for Google's experiment

"Although the median connection latency only increased by a millisecond, the latency for the slowest 5% increased by 20ms and, for the slowest 1%, by 150ms. Since NewHope is computationally inexpensive, we're assuming that this is caused entirely by the increased message sizes. Since connection latencies compound on the web (because subresource discovery is delayed), the data requirement of NewHope is moderately expensive for people on slower connections."

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Back to the drawing board!

KYBER

THE KEM

Shi Bai	Joppe Bos	Léo Ducas
Eike Kiltz	Tancrède Lepoint	Vadim Lyubashevsky
John M. Schanck	Peter Schwabe	Damien Stehlé

The design of Kyber (WiP)

- Use **Module-Lattices** and MLWE
  - RLWE: large polynomials (e.g., $n = 1024$)
  - LWE: matrices of integers with large dimension (e.g., 752×752, 752×8)
  - MLWE: matrices of smaller polynomials (e.g., $n = 256$) of small dimension (e.g., 3×3, 3×1)
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  - Possibility for further compression of keys and ciphertext (WiP)
- Easy to scale security by changing d

Kyber's encryption scheme

$q = 7681,\ n = 256,\ d = 3$

We work with matrices of polynomials in $\mathbb{Z}_{7681}[x]/(x^{256}+1)$ of dim. $d = 3$ and a distribution of poly with binomial coeffs. Ψ_4

KeyGen():

- seed$\leftarrow\{0,\ldots,255\}^{32}$
- $\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \leftarrow \text{SHAKE(seed)}$
- $\mathbf{s}, \mathbf{e} \leftarrow \Psi_4^d$
- $\mathbf{b} = \mathbf{A} \cdot \mathbf{s} + \mathbf{e}$
- Define $\text{pk} = (\text{seed}, \mathbf{b})$ and $\text{sk} = \mathbf{s}$

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Encrypt(pk, $m \in \{0,1\}^{256}$, coins):

- seed, $\mathbf{b} \leftarrow$ pk
- $\mathbf{A} = \text{SHAKE(seed)}$
- $\mathbf{s}' \leftarrow \Psi_4^d(\text{coins}, 1)$
- $\mathbf{e}' \leftarrow \Psi_4^d(\text{coins}, 2)$
- $e'' \leftarrow \Psi_4(\text{coins}, 3)$
- $\mathbf{u} = (\mathbf{s}')^t \cdot \mathbf{A} + \mathbf{e}'$
- $v = \langle \mathbf{b}, \mathbf{s}' \rangle + e'' + \lfloor q/2 \rfloor \cdot \sum_i m_i x^i$
- Output $(\mathbf{u}, v)$

Decrypt(sk, $(\mathbf{u}, v)$):

- $w = v - \langle \mathbf{u}, \mathbf{s} \rangle$
- for $i \in \{0, \ldots, 255\}$,
  $$m_i \leftarrow \begin{cases} 1 & \text{if } w_i \in (\frac{q}{4}, \frac{3 \cdot q}{4}) \\ 0 & \text{otherwise} \end{cases}$$
- Output $(m_0, \ldots, m_{255})$

Idea of the CCA transformation

Alice (Server)		Bob (Client)
Gen():		Enc(seed, $\mathbf{b}$):
pk, sk$\leftarrow$KeyGen()		$x\leftarrow\{0,\dots,255\}^{32}$
seed, $\mathbf{b}\leftarrow$pk	$\xrightarrow{\text{seed},\mathbf{b}}$	$x\leftarrow$SHA3-256(x)
		k, coins$\leftarrow$SHA3-512(x)
	$\xleftarrow{\mathbf{u},v}$	$\mathbf{u}, v\leftarrow$Encrypt((seed, $\mathbf{b}$), x, coins)
Dec($\mathbf{s}$, ($\mathbf{u}$, v)):		
$x'\leftarrow$ Decrypt($\mathbf{s}$, ($\mathbf{u}$, v))		
k', $coins'\leftarrow$SHA3-512(x')		
$\mathbf{u}', v'\leftarrow$Encrypt((seed, $\mathbf{b}$), x', coins$'$)		
verify if $(\mathbf{u}', v') = (\mathbf{u}, v)$		

Additionally:

- Hash the public key into the coins
- Hash the ciphertext into the final key

Kyber performance guesstimates

	NewHope	Kyber
public-key bytes	1824	1088
ciphertext bytes	2048	1152
Gen cycles	258 246	296 544
Enc cycles	384 994	401 960
Dec cycles	86 280	469 872

- Cycles are for C reference implementation on Haswell
- Optimized implementations for Kyber will follow

Stay tuned

http://pq-crystals.org/kyber