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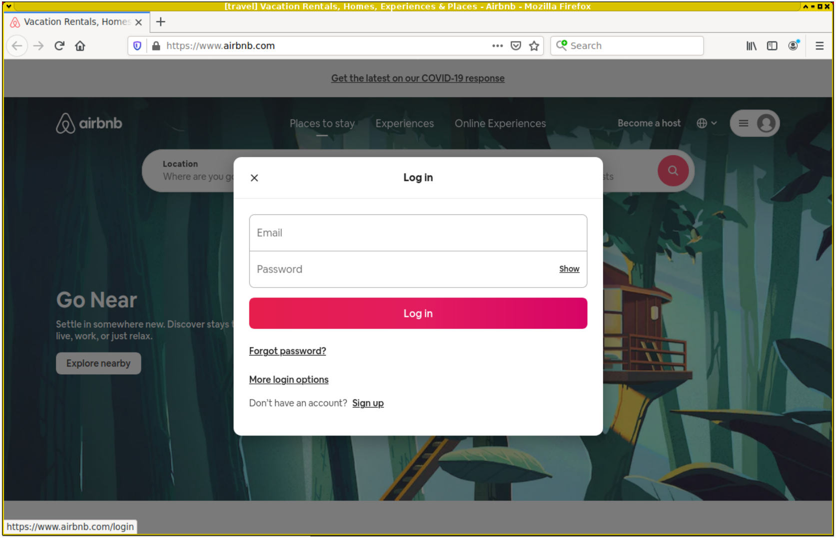
FOR SECURITY AND PRIVACY

Post-quantum key encapsulation: Kyber

Roberto Avanzi, Joppe Bos, Leo Ducas, Eike Kiltz, Tancrede Lepoint, Vadim Lyubashevsky, John M. Schanck, Peter Schwabe, Gregor Seiler, Damien Stehle, Jintai Ding

August 31, 2022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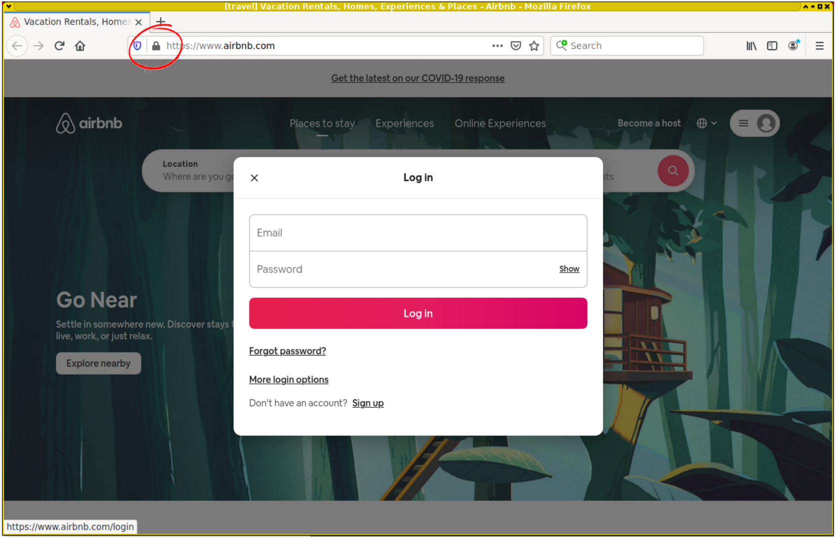
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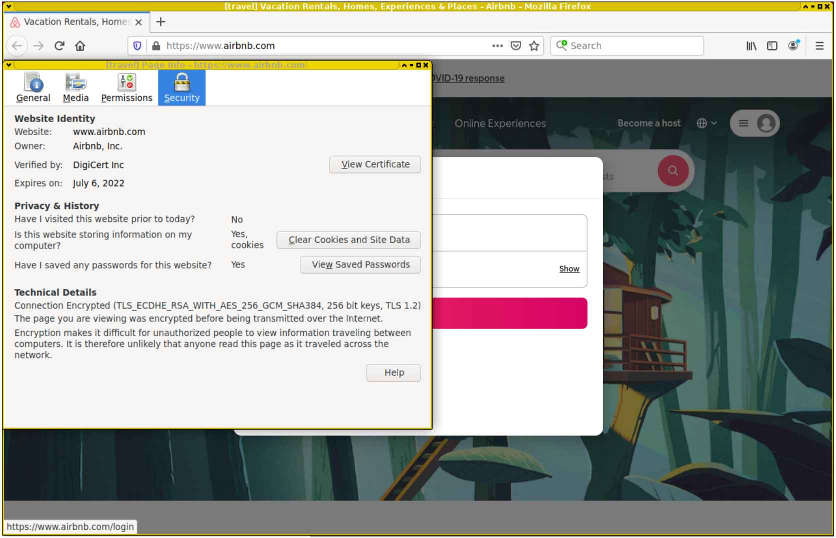
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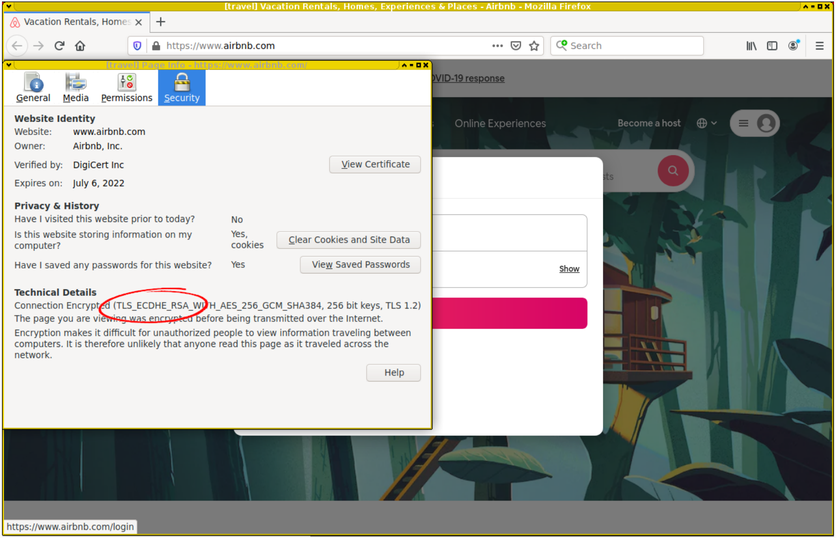
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View Saved Passwords
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Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer*

Peter W. Shor†

Abstract

A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

"In the past, people have said, maybe it's 50 years away, it's a dream, maybe it'll happen sometime. I used to think it was 50. Now I'm thinking like it's 15 or a little more. It's within reach. It's within our lifetime. It's going to happen."

—Mark Ketchen (IBM), Feb. 2012, about quantum computers

Post-quantum crypto

Definition

Post-quantum crypto is (asymmetric) crypto that resists attacks using classical *and quantum* computers.

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5 main directions

- Lattice-based crypto (PKE and Sigs)
- Code-based crypto (mainly PKE)
- Multivariate-based crypto (mainly Sigs)
- Hash-based signatures (only Sigs)
- Isogeny-based crypto (it's complicated. . .)

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- Selection through an open process and multiple rounds
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- PQC project:
  - Announcement: Feb 2016
  - Call for proposals: Dec 2016 (based on community input)
  - Deadline for submissions: Nov 2017

The NIST competition: initial overview

Count of Problem Category	Column Labels		
Row Labels	Key Exchange	Signature	Grand Total
?	1		1
Braids	1	1	2
Chebychev	1		1
Codes	19	5	24
Finite Automata	1	1	2
Hash		4	4
Hypercomplex Numbers	1		1
Isogeny	1		1
Lattice	24	4	28
Mult. Var	6	7	13
Rand. walk	1		1
RSA	1	1	2
Grand Total	**57**	**23**	**80**

4 31 27

Overview tweeted by Jacob Alperin-Sheriff on Dec 4, 2017.

The NIST competition, Feb 2019

Encryption / Key agreement

- 9 lattice-based
- 7 code-based
- 1 isogeny-based

Signature schemes

- 3 lattice-based
- 2 symmetric-crypto based
- 4 MQ-based

The NIST competition: Jul 2020

Finalists

- 4 key-agreement schemes
  - 3 lattice-based
  - 1 code-based
- 3 signature schemes
  - 2 lattice-based
  - 1 MQ-based

Alternate schemes

- 5 key-agreement schemes
  - 2 lattice-based
  - 2 code-based
  - 1 isogeny-based
- 3 signature schemes
  - 2 symmetric-crypto based
  - 1 MQ-based

The NIST competition: Aug 2022

4 schemes selected for standardization

- CRYSTALS-Kyber: lattice-based key agreement
- CRYSTALS-Dilithium: lattice-based signature
- Falcon: lattice-based signature
- SPHINCS$^+$: hash-based signature

4 schemes advanced to round 4

- Classic McEliece: code-based key agreement
- BIKE: code-based key agreement
- HQC: code-based key agreement
- SIKE: isogeny-based key agreement

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- SIKE: isogeny-based key agreement
- **Additionally:** call for more signature proposals

What now?

- Standards ready “by 2024”
- Time to start upgrading systems!

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Store now, decrypt later

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Let’s understand Kyber and what it means to use it.

A long time ago (2015) in a galaxy far, far away (Šibenik, Croatia)....

What is a Key Encapsulation Mechanism (KEM)?

Initiator | **Responder**

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KEM.Gen}$

$$\xrightarrow{\qquad\qquad\quad \mathsf{pk} \qquad\qquad\quad}$$

$(\mathsf{ct}, K) \leftarrow \mathsf{KEM.Enc}(\mathsf{pk})$

$$\xleftarrow{\qquad\qquad\quad \mathsf{ct} \qquad\qquad\quad}$$

$K \leftarrow \mathsf{KEM.Dec}(\mathsf{ct}, \mathsf{sk})$

Ring learning with errors (RLWE)

- Given $\mathbf{a}$, uniformly random
- Given “noise distribution” χ
- Given samples $\mathbf{as} + \mathbf{e}$, with $\mathbf{e} \leftarrow \chi$

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- Given $\mathbf{a}$, uniformly random
- Given “noise distribution” χ
- Given samples $\mathbf{as} + \mathbf{e}$, with $\mathbf{e} \leftarrow \chi$
- Search version: find $\mathbf{s}$
- Decision version: distinguish from uniform random

Where do $\mathbf{a}$, $\mathbf{e}$, and $\mathbf{s}$ live?

Short answer

In $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$

Where do $\mathbf{a}$, $\mathbf{e}$, and $\mathbf{s}$ live?

Longer answer

Polynomials with n coefficients, each coefficient in $\{0, \ldots, q-1\}$

Arithmetic uses reduction modulo q and modulo $(X^n + 1)$

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Example

Let $q = 7$ and $n = 4$.

Let $\mathbf{a} = (4X^3 + 5X^2 + 2X + 2)$ and $\mathbf{b} = (6X^3 + 4X^2 + 3)$

Where do $\mathbf{a}$, $\mathbf{e}$, and $\mathbf{s}$ live?

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Let $\mathbf{a} = (4X^3 + 5X^2 + 2X + 2)$ and $\mathbf{b} = (6X^3 + 4X^2 + 3)$

$$\begin{aligned} \mathbf{a} + \mathbf{b} &= 10X^3 + 9X^2 + 2X + 5 \\ &= 3X^3 + 2X^2 + 2X + 5 \end{aligned}$$

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$$\begin{aligned}\mathbf{a} - \mathbf{b} &= -2X^3 + X^2 + 2X - 1 \\ &= 5X^3 + X^2 + 2X + 6\end{aligned}$$

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$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} = \quad & 24X^6 + 16X^5 + 12X^3 + 30X^5 + 20X^4 + 15X^2 + \\ & 12X^4 + 8X^3 + 6X + 12X^3 + 8X^2 + 6\end{aligned}$$

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= \quad & -X^2 - 4X + 4X^3 + 2 \\
= \quad & 4X^3 + 6X^2 + 3X + 2
\end{aligned}$$

How to build a KEM: the basic idea

Alice (server)		Bob (client)
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{\mathbf{b}}$	$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
	$\xleftarrow{\mathbf{u}}$	

Alice has $\mathbf{v} = \mathbf{u}\mathbf{s} = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}'\mathbf{s}$

Bob has $\mathbf{v}' = \mathbf{b}\mathbf{s}' = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}\mathbf{s}'$

- Secret and noise polynomials $\mathbf{s}, \mathbf{s}', \mathbf{e}, \mathbf{e}'$ are small
- $\mathbf{v}$ and $\mathbf{v}'$ are *approximately* the same

How to build a KEM: the construction

Alice		Bob
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}\quad)}$	
		$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{b}\mathbf{s}'$
$\mathbf{v}' \leftarrow \mathbf{u}\mathbf{s}$	$\xleftarrow{(\mathbf{u}\quad)}$	

How to build a KEM: the construction

Alice		Bob
seed $\xleftarrow{\$} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow$ Parse(XOF(*seed*))		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, \mathit{seed})}$	$\mathbf{a} \leftarrow$ Parse(XOF(*seed*))
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
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		$\mathbf{v} \leftarrow \mathbf{bs}'$
		$k \xleftarrow{\$} \{0,1\}^n$
		$\mathbf{k} \leftarrow \text{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$

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$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$
$\mathbf{k}' \leftarrow \mathbf{c} - \mathbf{v}'$		$\mu \leftarrow \text{Extract}(\mathbf{k})$
$\mu \leftarrow \text{Extract}(\mathbf{k}')$		

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$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, \textit{seed})}$	$\mathbf{a} \leftarrow$ Parse(XOF(*seed*))
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}' + \mathbf{e}''$
		$k \xleftarrow{\$} \{0,1\}^n$
		$\mathbf{k} \leftarrow$ Encode(k)
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$
$\mathbf{k}' \leftarrow \mathbf{c} - \mathbf{v}'$		$\mu \leftarrow$ Extract($\mathbf{k}$)
$\mu \leftarrow$ Extract($\mathbf{k}'$)		

This is LPR encryption, written as KEM (except for generation of $\mathbf{a}$)

Encode and Extract

- Encoding in LPR encryption: map n bits to n coefficients:
    - A zero bit maps to 0
    - A one bit maps to $q/2$
- Idea: Noise affects low bits of coefficients, put data into high bits

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  - A zero bit maps to 0
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- Idea: Noise affects low bits of coefficients, put data into high bits
- Decode: map coefficient into $[-q/2, q/2]$
  - Closer to 0 (i.e., in $[-q/4, q/4]$): set bit to zero
  - Closer to $\pm q/2$: set bit to one

Two more steps to Kyber

MLWE instead of RLWE

IND-CCA2 Security

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- Easily scale security
- Optimized routines the same for all security levels

IND-CCA2 Security

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- Optimized routines the same for all security levels

IND-CCA2 Security

- Support static (or cached) keys
- More robust
- Useful for authenticated key exchange
- Easy to construct PKE

Module Learning with Errors (MLWE)

- RLWE uses arithmetic on large degree polynomials
- For example, NewHope uses $n = 1024$, $q = 12289$

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- Kyber: $n = 256$, $q = 3329$
  - Security level 1 (AES-128): $d = 2$
  - Security level 3 (AES-192): $d = 3$
  - Security level 5 (AES-256): $d = 4$
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- Noise is centered binomial $\mathrm{HW}(x) - \mathrm{HW}(y)$ for 2-bit x and y

Chosen-ciphertext attacks

- Decryption failures are a function of $\mathbf{s}$, $\mathbf{e}$, $\mathbf{s}'$, $\mathbf{e}'$
- Attacker can choose larger secret/noise $\mathbf{e}'$ and $\mathbf{s}'$
- Observe if decryption fails
- Learn something about $\mathbf{s}$

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- This is a chosen ciphertext attack (CCA)
- Learn full $\mathbf{s}$ after a few thousand queries
- NewHope never claimed CCA-security!
- This "attack" is completely expected
- Not a problem for ephemeral $\mathbf{s}$

From passive to CCA security

The Fujisaki-Okamoto Transform (idea)

- Build CCA-secure KEM from passively secure encryption scheme
- Make failure probability negligible for honest $\mathbf{s}'$, $\mathbf{e}'$, $\mathbf{e}''$
- Force encapsulator to generate $\mathbf{s}'$, $\mathbf{e}'$, $\mathbf{e}''$ honestly

From passive to CCA security

The Fujisaki-Okamoto Transform

Alice (Server)		Bob (Client)
Gen():		Encaps(pk):
pk, sk $\leftarrow$ KeyGen()	$\xrightarrow{\text{pk}}$	$x \leftarrow \{0, \ldots, 255\}^{32}$ k, coins $\leftarrow$ SHA3-512(x)
	$\xleftarrow{\text{ct}}$	ct $\leftarrow$ Encrypt(pk, x, coins)
Decaps((sk, pk), ct): $x' \leftarrow$ Decrypt(sk, ct) k', $coins'$ $\leftarrow$ SHA3-512(x') ct$'$ $\leftarrow$ Encrypt(pk, x', coins$'$) **verify if** ct $=$ ct$'$		

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Additionally in Kyber:

- Hash the (hash of the) public key into x
  - Multi-target protection (for coins)
  - Turn into contributory KEM
- Hash the (hash of the) ciphertext into the final key

Kyber for Engineers, the baseline

Key exchange today: ECDH

- Key-pair generation $\approx 100,000$ Comet Lake cycles
- Shared-key computation $\approx 100,000$ Comet Lake cycles
- Public keys have 32 bytes

Kyber for Engineers, part I: A KEM is not DH!

Alice		Bob
$A \leftarrow g^a$		$B \leftarrow g^b$
	$\xrightarrow{\qquad\qquad A \qquad\qquad}$	
	$\xleftarrow{\qquad\qquad B \qquad\qquad}$	
$K \leftarrow B^a = (g^b)^a = g^{ab}$		$K \leftarrow A^b = (g^a)^b = g^{ab}$

Kyber for Engineers, part I: A KEM is not DH!

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	B	
	A	
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Kyber for Engineers, part I: A KEM is not DH!

Initiator | **Responder**

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KEM.Gen}$

$\xrightarrow{\quad \mathsf{pk} \quad}$

$(\mathsf{ct}, K) \leftarrow \mathsf{KEM.Enc}(\mathsf{pk})$

$\xleftarrow{\quad \mathsf{ct} \quad}$

$K \leftarrow \mathsf{KEM.Dec}(\mathsf{ct}, \mathsf{sk})$

Kyber for Engineers, part II: Performance

Kyber768 (NIST Security level 3)

- Key-pair generation $\approx 40,000$ Comet Lake cycles
- Encapsulation $\approx 55,000$ Comet Lake cycles
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- Public keys have 1184 bytes
- Ciphertexts have 1088 bytes
- Cycles are dominated by Keccak!

- FO-transform: hide if decryption succeeded
- Use full re-encryption to do this

Kyber for Engineers, part III: SCA and FI against FO

- FO-transform: hide if decryption succeeded
- Use full re-encryption to do this
- Long computation, one bit of information
- Very hard to protect against SCA/FI

Recommendations

- Start playing with Kyber
- Assume that details may still change

Recommendations

- Start playing with Kyber
- Assume that details may still change
- Always combine with pre-quantum crypto (hybrid KEMs)
- Use Kyber768 (or Kyber1024)

Online references

- NIST PQC website:
  https://csrc.nist.gov/Projects/Post-Quantum-Cryptography
- NIST mailing list:
  https://csrc.nist.gov/projects/post-quantum-cryptography/email-list
  https://groups.google.com/a/list.nist.gov/g/pqc-forum
- Kyber:
  https://pq-crystals.org/kyber
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