POST-QUANTUM KEY EXCHANGE

A NEW HOPE

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Quantum computers – should we be worried?

"In the past, people have said, maybe it's 50 years away, it's a dream, maybe it'll happen sometime. I used to think it was 50. Now I'm thinking like it's 15 or a little more. It's within reach. It's within our lifetime. It's going to happen."

—Mark Ketchen (IBM), Feb. 2012, about quantum computers

Quantum computers – should we be worried?

“Now, we’re aiming to build the first quantum integrated circuit, which we’re aiming for by 2020.

Beyond that, we must do error correction, so that if errors come into the chip, you can run multiple processes in parallel to eliminate those errors and that error correction will take another five years or so.”

—Michelle Simmons (UNSW), Jan. 2016

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  Need post-quantum (ephemeral) encryption keys now!
- Combine with pre-quantum key exchange to not lower security

Ring-Learning-with-errors (RLWE)

- Let $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Let χ be an *error distribution* on $\mathcal{R}_q$
- Let $\mathbf{s} \in \mathcal{R}_q$ be secret
- Attacker is given pairs $(\mathbf{a}, \mathbf{as} + \mathbf{e})$ with
  - $\mathbf{a}$ uniformly random from $\mathcal{R}_q$
  - $\mathbf{e}$ sampled from χ
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- Common choice for χ: discrete Gaussian
- Common optimization for protocols: fix $\mathbf{a}$

Peikert's RLWE-based KEM

Parameters: q, n, χ

KEM.Setup() :

$\mathbf{a} \xleftarrow{\$} \mathcal{R}_q$

Alice (server)		Bob (client)
KEM.Gen($\mathbf{a}$) :		KEM.Encaps($\mathbf{a}, \mathbf{b}$) :
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{\mathbf{b}}$	$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}' + \mathbf{e}''$
		$\bar{\mathbf{v}} \xleftarrow{\$} \mathsf{dbl}(\mathbf{v})$
KEM.Decaps($\mathbf{s}, (\mathbf{u}, \mathbf{v}')$) :	$\xleftarrow{\mathbf{u}, \mathbf{v}'}$	$\mathbf{v}' = \langle \bar{\mathbf{v}} \rangle_2$
$\mu \leftarrow \mathsf{rec}(2\mathbf{us}, \mathbf{v}')$		$\mu \leftarrow \lfloor \bar{\mathbf{v}} \rceil_2$

BCNS key exchange

- Bos, Costello, Naehrig, Stebila, IEEE S&P 2015:
  - Phrase the KEM as key exchange
  - Instantiate with concrete parameters
  - Integrate with OpenSSL $\rightarrow$ post-quantum TLS key exchange
  - Also: combined ECDH+RLWE key exchange

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  - Also: combined ECDH+RLWE key exchange
- Parameters chosen by BCNS:
  - $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
  - $n = 1024$
  - $q = 2^{32} - 1$
  - $\chi = D_{\mathbb{Z},\sigma}$
  - $\sigma = 8\sqrt{2\pi} \approx 3.192$

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  - $n = 1024$
  - $q = 2^{32} - 1$
  - $\chi = D_{\mathbb{Z},\sigma}$
  - $\sigma = 8\sqrt{2\pi} \approx 3.192$
- Claimed security level: 128 bits pre-quantum
- Failure probability: $\approx 2^{-131072}$

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- Encode polynomials in NTT domain
- Multiple implementations

A new hope – protocol

Parameters: $q = 12289 < 2^{14},\ n = 1024$

Error distribution: ψ_{16}

Alice (server)		**Bob (client)**
$seed \xleftarrow{\$} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \text{Parse}(\text{SHAKE-128}(seed))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \psi_{16}^n$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \psi_{16}^n$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, seed)}$	$\mathbf{a} \leftarrow \text{Parse}(\text{SHAKE-128}(seed))$
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
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$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{r})}$	$\mathbf{r} \xleftarrow{\$} \text{HelpRec}(\mathbf{v})$
$k \leftarrow \text{Rec}(\mathbf{v}', \mathbf{r})$		$k \leftarrow \text{Rec}(\mathbf{v}, \mathbf{r})$
$\mu \leftarrow \text{SHA3-256}(k)$		$\mu \leftarrow \text{SHA3-256}(k)$

Error reconciliation

- After running the protocol
  - Alice has $\mathbf{x}_A = \mathbf{ass}' + \mathbf{e}'\mathbf{s}$
  - Bob has $\mathbf{x}_B = \mathbf{ass}' + \mathbf{es}' + \mathbf{e}''$
- Those elments are similar, but not the same
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- Newhope: extract one bit from multiple coefficients (increase security)
- Specifically: 1 bit from 4 coefficients $\rightarrow$ 256-bit key from 1024 coefficients

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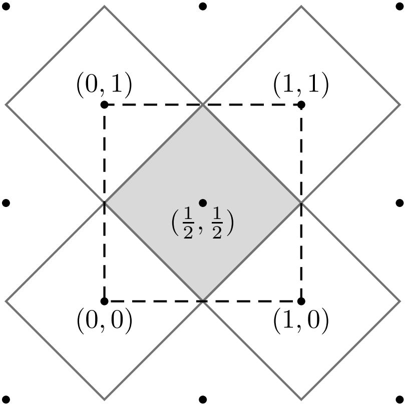
- In the following: 2-dimensional intuition (4-dim. case very similar)
- "Scale" vector $\mathbf{x}$ to $[0, 1)^2$

2D Error reconciliation



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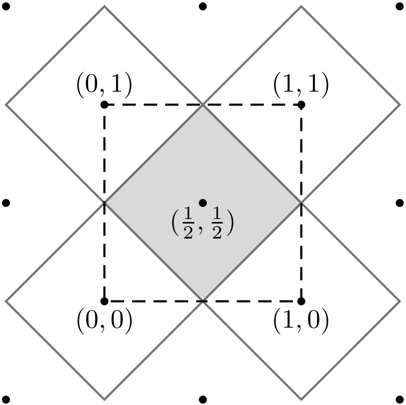

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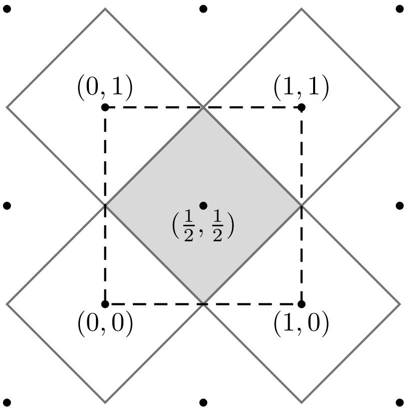
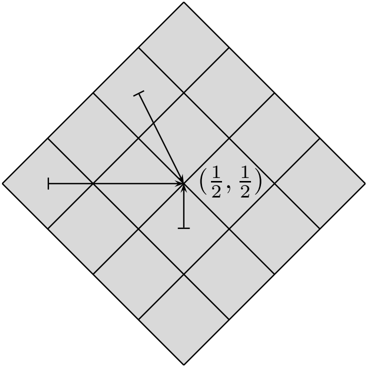


- If $\mathbf{x}$ is in the grey Voronoi cell: pick key bit 1
- If $\mathbf{x}$ is in the white Voronoi cell: pick key bit 0
- Reconciliation: Bob sends difference vector from $\mathbf{x}_B$ to center of his Voronoi cell
- Alice adds this difference vector to her vector $\mathbf{x}_A$

Discretization of reconciliation



- Sending difference vector means doubling communcation
- Idea: chop Voronoi cell into 2^{dr} subcells
  - d: dimension (4 for NewHope)
  - r: discretization level
- Need to send only rd bits per d coefficients
- NewHope: $r = 2$; hence 256 bytes of reconciliation information

“Blurring the edges”

- This would all work if $\mathbf{x}$ was continuous uniform from $[0, 1)$
- We start with $\mathbf{x} \in \{0, \ldots, q-1\}^2$, q odd
- Odd number of possible values; no way to pick key bit without bias!
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- This is a generalization of Peikert’s “randomized doubling” trick

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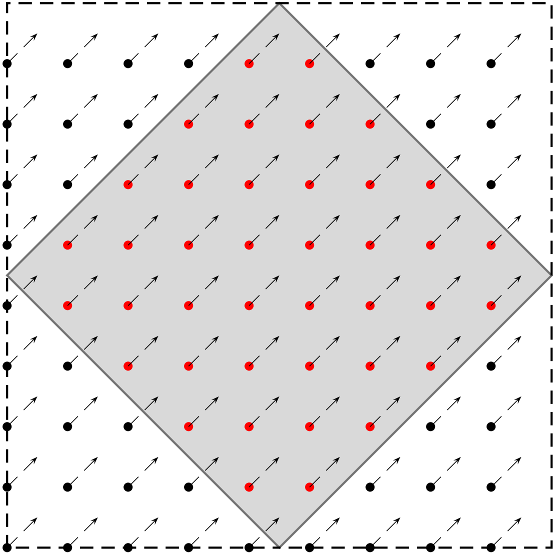

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- Dual attack: find short vector in dual lattice
- Length determines complexity and attacker's advantage ϵ

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- Use root lattice D_2 instead of D_4
- Use $k = 24$ for the centered binomial distribution

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JarJar is not recommended for use!

Post-quantum security

Attack	m	b	Known Classical	Known Quantum	Best Plausible
BCNS proposal: $q = 2^{32} - 1$, $n = 1024$, $\sigma = 3.192$					
Primal	1062	296	86	77	61
Dual	1042	259	84	**76**	62
JarJar: $q = 12289$, $n = 512$, $\sigma = \sqrt{12}$					
Primal	623	449	131	117	93
Dual	531	341	106	**95**	77
NewHope: $q = 12289$, $n = 1024$, $\sigma = \sqrt{8}$					
Primal	1100	967	282	253	200
Dual	938	761	229	**206**	165

- b: Block size for BKZ
- m: Number of used samples

Against all authority

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- What if $\mathbf{a}$ is backdoored?
- Parameter-generating authority can break key exchange
- "Solution": Nothing-up-my-sleeves (involves endless discussion!)

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- **Must not reuse keys/noise!**

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- AVX2 implementation:
  - Speed up NTT using vectorized `double` arithmetic
  - Use AES-256 for noise sampling
  - Use AVX2 for centered binomial

The protocol revisited

Parameters: $q = 12289 < 2^{14},\ n = 1024$

Error distribution: ψ_{16}^n

Alice (server)		Bob (client)
$seed \xleftarrow{\$} \{0, \ldots, 255\}^{32}$		
$\hat{\mathbf{a}} \leftarrow \mathsf{Parse}(\mathsf{SHAKE\text{-}128}(seed))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \psi_{16}^n$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \psi_{16}^n$
$\hat{\mathbf{s}} \leftarrow \mathsf{NTT}(\mathbf{s})$		
$\hat{\mathbf{b}} \leftarrow \hat{\mathbf{a}} \circ \hat{\mathbf{s}} + \mathsf{NTT}(\mathbf{e})$	$\xrightarrow[1824\ \mathsf{Bytes}]{m_a = \mathsf{encodeA}(seed, \hat{\mathbf{b}})}$	$(\hat{\mathbf{b}}, seed) \leftarrow \mathsf{decodeA}(m_a)$
		$\hat{\mathbf{a}} \leftarrow \mathsf{Parse}(\mathsf{SHAKE\text{-}128}(seed))$
		$\hat{\mathbf{t}} \leftarrow \mathsf{NTT}(\mathbf{s}')$
		$\hat{\mathbf{u}} \leftarrow \hat{\mathbf{a}} \circ \hat{\mathbf{t}} + \mathsf{NTT}(\mathbf{e}')$
		$\mathbf{v} \leftarrow \mathsf{NTT}^{-1}(\hat{\mathbf{b}} \circ \hat{\mathbf{t}}) + \mathbf{e}''$
$(\hat{\mathbf{u}}, \mathbf{r}) \leftarrow \mathsf{decodeB}(m_b)$	$\xleftarrow[2048\ \mathsf{Bytes}]{m_b = \mathsf{encodeB}(\hat{\mathbf{u}}, \mathbf{r})}$	$\mathbf{r} \xleftarrow{\$} \mathsf{HelpRec}(\mathbf{v})$
$\mathbf{v}' \leftarrow \mathsf{NTT}^{-1}(\hat{\mathbf{u}} \circ \hat{\mathbf{s}})$		$k \leftarrow \mathsf{Rec}(\mathbf{v}, \mathbf{r})$
$k \leftarrow \mathsf{Rec}(\mathbf{v}', \mathbf{r})$		$\mu \leftarrow \mathsf{SHA3\text{-}256}(k)$
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Vector computations

Scalar computation

- Load 32-bit integer a
- Load 32-bit integer b
- Perform addition $c \leftarrow a + b$
- Store 32-bit integer c

Vectorized computation

- Load 8 consecutive 32-bit integers $(a_0, a_1, \ldots, a_7)$
- Load 8 consecutive 32-bit integers $(b_0, b_1, \ldots, b_7)$
- Perform addition $(c_0, c_1, \ldots, c_7) \leftarrow (a_0 + b_0, a_1 + b_1, \ldots, a_7 + b_7)$
- Store 256-bit vector $(c_0, c_1, \ldots, c_7)$

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- Situation on other architectures/microarchitectures is similar

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- Strong synergies between speeding up code with vector instructions and protecting code!

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- TODO: Revisit NTT, use integer arithmetic

Performance

	BCNS	**C ref**	**AVX2**
Key generation (server)	$\approx 2\,477\,958$	$271\,650$ $(272\,174)$	$115\,414$ $(115\,746)$
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- Also optimize other building blocks of NewHope

The NTT (in C)

- 10 levels of butterfly transformations
- Each level uses 512 Gentleman-Sande Butterflies

```
W = omega[jTwiddle++]; // W is in Montgomery domain
t = a[j];
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- Every second level omits the `barrett_red`

Montgomery reduction

```
uint16_t montgomery_red(uint32_t a) {
  uint32_t u;
  u = (a * 12287);
  u &= ((1 << 18) - 1);
  a += u * 12289;
  return a >> 18;
}
```

- Use Montgomery parameter $R = 2^{18}$
- Works for inputs in $\{0, \ldots, 2^{32} - q(R-1) - 1\}$
- Output is guaranteed to have at most 14 bits

Short Barrett reduction

```
uint16_t barrett_red(uint16_t a) {
  uint32_t u;
  u = ((uint32_t) a * 5) >> 16;
  a -= u * 12289;
  return a;
}
```

- Accepts any 16-bit unsigned int as input
- Produces outputs of at most 14 bits

ARM Cortex-M0 results

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- How about Nussbaumer's algorithm for multiplication?
- How about Karatsuba + Toom for multiplication?
- How about smaller n (e.g., $n \approx 800$)?

NewHope online

Paper:	`https://cryptojedi.org/papers/#newhope`
Software:	`https://cryptojedi.org/crypto/#newhope`
ARM software:	`https://github.com/newhopearm/newhopearm.git`
Newhope in Go:	`https://github.com/Yawning/newhope` (by Yawning Angel)
Newhope in Rust:	`https://code.ciph.re/isis/newhopers` (by Isis Lovecruft)
Newhope in D:	`https://cryptojedi.org/crypto/#newhope` (?) (soon???)