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The NIST post-quantum project

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5 building blocks for a "secure channel"
Symmetric crypto

- Block or stream cipher (e.g., AES, ChaCha20)
- Authenticator (e.g., HMAC, GMAC, Poly1305)
- Hash function (e.g., SHA-2, SHA-3)

Crypto today

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The asymmetric monoculture

- All widely deployed asymmetric crypto relies on
  - the **hardness of factoring**, or
  - the **hardness of (elliptic-curve) discrete logarithms**

Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer*

Peter W. Shor†

Abstract

A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

Will there be quantum computers?

"In the past, people have said, maybe it's 50 years away, it's a dream, maybe it'll happen sometime. I used to think it was 50. Now I'm thinking like it's 15 or a little more. It's within reach. It's within our lifetime. It's going to happen."

—Mark Ketchen (IBM), Feb. 2012, about quantum computers

Definition

Post-quantum crypto is (asymmetric) crypto that resists attacks using classical *and quantum* computers.

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5 main directions

- Lattice-based crypto (PKE and Sigs)
- Code-based crypto (mainly PKE)
- Multivariate-based crypto (mainly Sigs)
- Hash-based signatures (only Sigs)
- Isogeny-based crypto (so far, mainly PKE)

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- PQC project:
  - Announcement: Feb 2016
  - Call for proposals: Dec 2016 (based on community input)
  - Deadline for submissions: Nov 2017

The NIST PQC “not-a-competition” ctd.

Submission categories

- Cryptographic signatures (only stateless)
  - Security for at least 2^{64} signatures per key
- Public-key encryption / key encapsulation
  - Passive or active security (CPA or CCA2)

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Security categories

- Level 1: Equivalent to AES-128 (pre- and post-quantum)
- Level 2: Equivalent to SHA-256 (pre- and post-quantum)
- Level 3: Equivalent to AES-192 (pre- and post-quantum)
- Level 4: Equivalent to SHA-512 (pre- and post-quantum)
- Level 5: Equivalent to AES-256 (pre- and post-quantum)

The NIST competition, initial overview

Count of Problem Category	Column Labels		
Row Labels	Key Exchange	Signature	Grand Total
?	1		1
Braids	1	1	2
Chebychev	1		1
Codes	19	5	24
Finite Automata	1	1	2
Hash		4	4
Hypercomplex Numbers	1		1
Isogeny	1		1
Lattice	24	4	28
Mult. Var	6	7	13
Rand. walk	1		1
RSA	1	1	2
Grand Total	**57**	**23**	**80**

Overview tweeted by Jacob Alperin-Sheriff on Dec 4, 2017.

"Key exchange"

- What is meant is **key encapsulation mechanisms** (KEMs)
  - $(\mathrm{vk}, \mathrm{sk}) \leftarrow \mathrm{KeyGen}()$
  - $(c, k) \leftarrow \mathrm{Encaps}(\mathrm{vk})$
  - $k \leftarrow \mathrm{Decaps}(c, \mathrm{sk})$

The NIST competition (ctd.)

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Status of the NIST competition

- In total 69 submissions accepted as "complete and proper"
- Several broken, 5 withdrawn
- Jan 2019: NIST announces 26 round-2 candidates
  - 17 KEMs and PKEs
  - 9 signature schemes

NIST finalists as drop-in replacements?

- Can wait until NIST standardizes some algorithms in ≈ 5 years
- Plug these algorithms into existing protocols and systems
- My impression: that's what many systems designers expect

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- Would generate a generation of rather poor protocols
  - mediocre performance (designed pre-quantum, instantiated post-quantum)
  - Suboptimal security properties

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- Would generate a generation of rather poor protocols
  - mediocre performance (designed pre-quantum, instantiated post-quantum)
  - Suboptimal security properties
- Bad crypto is very hard to get rid of (think MD5)
- We probably have *one shot* to get this done properly
  - Systems *will* have to transition to PQ crypto
  - Let's work on getting the best out of this transition!
  - Requires interaction between cryptographers and systems designers

The starting point: ECC

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  - Keys and ciphertexts: 32 bytes
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- Let's look at post-quantum candidates (at NIST security level 3)

PQ-Signatures, part 1: $\mathcal{MQ}$-based

- Find solution to system of m quadratic eqns in n variables over $\mathbb{F}_q$
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- Example: NIST candidate GeMSS (others: Rainbow, LUOV)
  - Signing: ≈ 2.7 billion cycles
  - Verification: $\approx 580\,000$ cycles
  - Signature: ≈ 50 bytes
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- Can also construct signatures *with* reduction from $\mathcal{MQ}$
- Example: NIST candidate MQDSS
  - Signing $\approx$ 15 Mio cycles
  - Verification $\approx$ 10 Mio cycles
  - Signature: $\approx$ 60 KB
  - Public key: 64 bytes

PQ-Signatures, part 2: lattice-based

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- Based on, e.g., LWE (see later) and SIS
- All NIST candidates use structured lattices (again, see later)
- Example: Dilithium (others: qTESLA, FALCON)
  - Signing: $\approx 500\,000$ cycles
  - Verification: $\approx 170\,000$ cycles
  - Public key: $\approx 1.5\,\text{KB}$
  - Signature: $\approx 2.7\,\text{KB}$

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- Example: SPHINCS$^+$-SHA256-192f-robust
  - Signing: $\approx$ 66 Mio cycles
  - Verification: $\approx$ 9.6 Mio cycles
  - Signature: $\approx$ 35.5 KB
  - Public key: 48 bytes
  - Up to 2^{64} signatures

PQ-KEMs, part 1: code-based

- Idea: Take error-correcting code for up to t errors
- Keep *decoding* algorithm secret
- Encryption: map message to code word, add t errors
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- Most prominent example: McEliece (1978), uses binary Goppa codes
- "Classic McEliece" KEM NIST submission (other: NTS-KEM)
  - Encapsulation: $\approx 90\,000$ cycles
  - Decapsulation: $\approx 270\,000$ cycles
  - Key generation: ≈ 300 Mio cycles
  - Cipher text: 188 bytes
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- Probably good choice for, e.g., GPG, but not for low-latency applications
- Possible solution: use structured codes
  (**NIST candidates**: BIKE, LEDAcrypt, HQC, ROLLO, RQC)
- Less studied, less conservative, often problems with CCA security

PQ-KEMs, part 3: SIKE

- Started as "supersingular-isogeny Diffie-Hellman" (SIDH), Jao, De Feo, 2011
- Given two elliptic curves E, E' from the same isogeny class
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  - Even more compact (and slower) with compression

Lattice-based KEMs

- 9 out of 19 NIST round-2 KEMs are (sort of) lattice based:
    - CRYSTALS-Kyber (short: Kyber)
    - FrodoKEM
    - LAC
    - NewHope
    - NTRU
    - NTRU Prime
    - Round5
    - Saber
    - Threebears
- I'm involved in CRYSTALS-Kyber, NewHope, and NTRU
- Two main reasons for the large number:
    - Large design space with many tradeoffs
    - Popularity before the NIST project (in particular through NewHope)

Google Security Blog

The latest news and insights from Google on security and safety on the Internet

Experimenting with Post-Quantum Cryptography

July 7, 2016

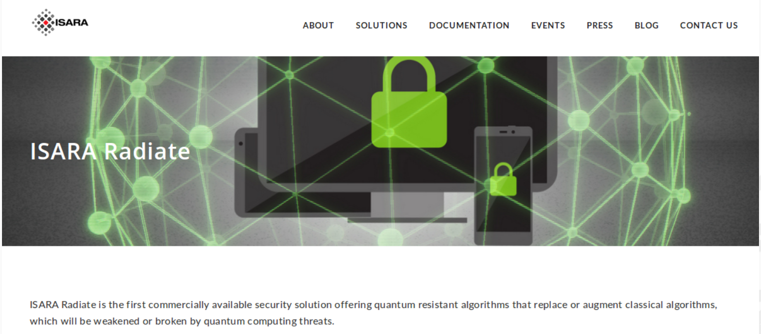
Posted by Matt Braithwaite, Software Engineer

Search blog ...

Archive

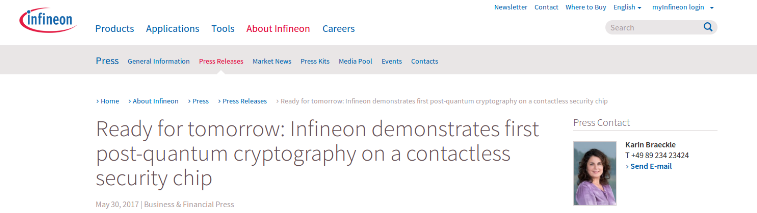
“We’re indebted to Erdem Alkim, Léo Ducas, Thomas Pöppelmann and Peter Schwabe, the researchers who developed “New Hope”, the post-quantum algorithm that we selected for this experiment.”

https://security.googleblog.com/2016/07/experimenting-with-post-quantum.html



"Key Agreement using the 'NewHope' lattice-based algorithm detailed in the New Hope paper, and LUKE (Lattice-based Unique Key Exchange), an ISARA speed-optimized version of the NewHope algorithm."

https://www.isara.com/isara-radiate/



"The deployed algorithm is a variant of "New Hope", a quantum-resistant cryptosystem"

https://www.infineon.com/cms/en/about-infineon/press/press-releases/2017/INFCCS201705-056.html

Learning with errors (LWE)

- Given uniform $\mathbf{A} \in \mathbb{Z}_q^{k \times \ell}$
- Given "noise distribution" χ
- Given samples $\mathbf{As} + \mathbf{e}$, with $\mathbf{e} \leftarrow \chi$

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  - Kyber/Saber: use small-dimension matrices and vectors over $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^{256} + 1)$

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  - Kyber/Saber: use small-dimension matrices and vectors over $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^{256} + 1)$
- Perform arithmetic on (vectors of) polynomials instead of vectors/matrices over $\mathbb{Z}_q$

How to build a KEM?

Alice (server)		Bob (client)
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{\mathbf{b}}$	$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
	$\xleftarrow{\mathbf{u}}$	

$$\begin{array}{lllll} \text{Alice has} & \mathbf{v} & = \mathbf{u}\mathbf{s} & = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}'\mathbf{s} \\ \text{Bob has} & \mathbf{v}' & = \mathbf{b}\mathbf{s}' & = \mathbf{a}\mathbf{s}\mathbf{s}' + \mathbf{e}\mathbf{s}' \end{array}$$

- Secret and noise polynomials $\mathbf{s}, \mathbf{s}', \mathbf{e}, \mathbf{e}'$ are small
- $\mathbf{v}$ and $\mathbf{v}'$ are *approximately* the same

How to build a KEM, part 2

Alice		Bob
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}\quad)}$	
		$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{b}\mathbf{s}'$
$\mathbf{v}' \leftarrow \mathbf{u}\mathbf{s}$	$\xleftarrow{(\mathbf{u}\quad)}$	

Alice		Bob
$seed \xleftarrow{\$} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \mathrm{Parse}(\mathrm{SHAKE\text{-}128}(seed))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, seed)}$	$\mathbf{a} \leftarrow \mathrm{Parse}(\mathrm{SHAKE\text{-}128}(seed))$
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}'$
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		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}'$
		$k \xleftarrow{\$} \{0,1\}^n$
		$\mathbf{k} \leftarrow \text{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$

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$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, seed)}$	$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{SHAKE\text{-}128}(seed))$
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
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		$k \xleftarrow{\$} \{0,1\}^n$
		$\mathbf{k} \leftarrow \mathsf{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$

How to build a KEM, part 2

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This is LPR encryption, written as KEX (except for generation of **a**)

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- Fujisaki-Okamoto transform (sketched):

Alice (Server)		Bob (Client)
$\underline{\text{Gen}()}$:		$\underline{\text{Enc}(\text{seed}, \mathbf{b})}$:
$\text{pk}, \text{sk} \leftarrow \text{KeyGen}()$		$x \leftarrow \{0, \ldots, 255\}^{32}$
$\text{seed}, \mathbf{b} \leftarrow \text{pk}$	$\xrightarrow{\text{seed}, \mathbf{b}}$	$x \leftarrow \text{SHA3-256}(x)$
		$k, \text{coins} \leftarrow \text{SHA3-512}(x)$
	$\xleftarrow{\mathbf{u}, v}$	$\mathbf{u}, v \leftarrow \text{Encrypt}((\text{seed}, \mathbf{b}), x, \text{coins})$
$\underline{\text{Dec}(\mathbf{s}, (\mathbf{u}, v))}$:		
$x' \leftarrow \text{Decrypt}(\mathbf{s}, (\mathbf{u}, v))$		
$k', coins' \leftarrow \text{SHA3-512}(x')$		
$\mathbf{u}', v' \leftarrow \text{Encrypt}((\text{seed}, \mathbf{b}), x', \text{coins}')$		
verify if $(\mathbf{u}', v') = (\mathbf{u}, v)$		

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- Advantages/Disadvantages compared to LPR:
  - Asymptotically weaker than Ring-LWE approach
  - Slower keygen, but faster encryption/decryption

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- "Traditionally", work directly with elements of $\mathcal{R}_q$ ("Ring-LWE")
- Alternative: Module-LWE (MLWE):
  - Choose smaller n, e.g., $n = 256$ (Kyber, Saber, ThreeBears)
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- MLWE can very easily scale security (change dimension of matrix):
  - Optimize arithmetic in $\mathcal{R}_q$ once
  - Use same optimized $\mathcal{R}_q$ arithmetic for all security levels

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  - **Fixed-weight noise or not?**
    - Fixed-weight noise needs random permutation (sorting)
    - Naive implementations leak secrets through timing
    - Advantage of fixed-weight: easier to bound (or eliminate) decryption failures

Design space 4: allow failures?

- Can avoid decryption failures entirely (NTRU, NTRU Prime)
- Advantage:
  - Easier CCA security transform and analysis
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- Active (CCA) security needs negligible failure prob.

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  - Infeasible today, but who knows. . .
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- Solution in NewHope: Choose a fresh **a** every time
- Server can cache **a** for some time (e.g., 1h)
- All NIST PQC candidates now use this approach

Design space 6: error-correcting codes?

- Ring-LWE/LWR schemes work with polynomials of > 256 coefficients
- "Encrypt" messages of > 256 bits
- **Need to encrypt** only 256-bit key
- Question: How do we put those additional bits to use?
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- NewHope: very simple threshold decoding
- LAC, Round5: more advanced ECC
  - Correct more error, obtain smaller public key and ciphertext
  - More complex to implement, in particular without leaking through timing

Design space 7: CCA security?

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- **Advantages:**
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  - Simpler to implement, no CCA transform
- **Disadvantages:**
  - Less robust (will somebody reuse keys?)
  - More options (CCA vs. CPA): easier to make mistakes

Design space 8: CCA transforms

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- Tweaks to FO transform:
  - Hash public-key into coins: multitarget protection (for non-zero failure probability)

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- As of round 2, no proposal uses explicit rejection
  - Would break some security reduction
  - More robust in practice (return value alwas 0)

(Exercise) Resources

- Overview NIST round-2 candidates: https://csrc.nist.gov/Projects/Post-Quantum-Cryptography/round-2-submissions
- Slides from 2nd NIST standardization conference: https://csrc.nist.gov/Events/2019/Second-PQC-Standardization-Conference
- NIST PQC Wiki (Florida Atlantic University): https://pqc-wiki.fau.edu