FSBday:

Implementing Wagner's Generalized Birthday Attack against the round-1 SHA-3 Candidate FSB

Christiane Peters, Peter Schwabe
joint work with Dan Bernstein, Tanja Lange and Ruben Niederhagen

Eindhoven University of Technology

Technische Universiteit
Eindhoven
University of Technology

September 10, 2009

SHARCS 2009

Wagner's generalized birthday attack

Given 2^{i-1} lists containing B-bit strings.

Generalized birthday problem:
The 2^{i-1}-sum problem consists of finding 2^{i-1} elements — exactly one per list — such that their sum equals 0 (modulo 2).

Wagner (CRYPTO '02)

We can expect a solution to the generalized birthday problem after one run of an algorithm using time $O((i-1) \cdot 2^{B/i})$ and lists of size $O(2^{B/i})$.

Wagner's tree algorithm

Given 4 lists containing each about $2^{B/3}$ elements which are chosen uniform at random from $\{0,1\}^B$.

- On level 0 take two lists and compare their elements on their least significant $B/3$ bits.

  Merge: If two elements coincide on those $B/3$ bits; put the xor of both elements into a new list. Proceed in the same manner with the other two lists.

  Uniform randomness of the elements $\Rightarrow$ both lists will contain about $2^{B/3}$ elements.

- On level 1 take the remaining two lists. Compare their elements by considering the remaining $2B/3$ bits.

  Expect to get 1 match after the merge step.

Tree algorithm for 2^{i-1} lists

The tree algorithm generalizes to 2^{i-1} lists as follows:

- Compare lists — always two at a time — by looking at the least significant B/i bits of elements.

- On level $i-2$ we are left with two lists whose elements need to be compared on $2B/i$ remaining bits.

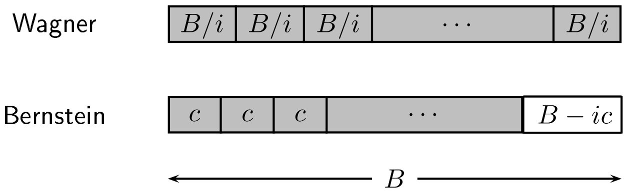
Precomputation step

Technische Universiteit **Eindhoven** University of Technology

Suppose that there is space for lists of size only 2^c with $c < B/i$.

Bernstein (SHARCS '07):

- Generate $2^{c\cdot(B-ic)}$ entries and only consider those of which the least significant $B - ic$ bits are zero.
- Then apply Wagner's algorithm with lists of size 2^c and clamp away c bits on each level.



Precomputation step

Suppose that there is space for lists of size only 2^c with $c < B/i$.

Bernstein (SHARCS '07):

- Generate $2^{c \cdot (B-ic)}$ entries and only consider those of which the least significant $B - ic$ bits are zero.
- Then apply Wagner's algorithm with lists of size 2^c and clamp away c bits on each level.

Generalization:

- The least significant $B - ic$ bits can have an arbitrary value
- Clamping value does not have to be the same on all lists (but: sum of all clamping values has to be 0).
- If an attack does not produce a collision we simply restart the attack with different clamping values.

Repeating (parts of) the tree algorithm

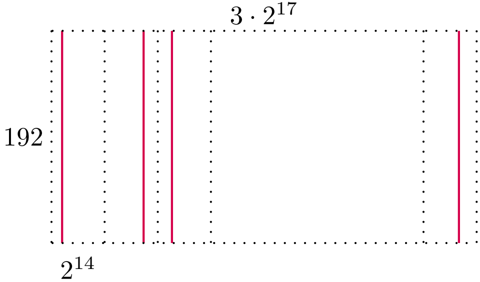
- When performing the algorithm with smaller lists some bits are left "uncontrolled" at the end.
- Deal with the lower success probability by repeatedly running the attack with different clamping values.
- We can apply the same idea of changing clamping values to an arbitrary merge step of the tree algorithm.

Target: the compression function of FSB_{48}

Given a binary random 192×393216 matrix H; number of blocks: $w = 24$.

Input: a regular weight-24 bit string of length 393216, i.e., there is exactly a single 1 in each interval $[(i-1)\cdot 16384, i \cdot 16834]_{1 \le i \le 24}$.

Output: Xor the 48 columns indicated by the input bit string.



Goal: Find a collision in FSB_{48}'s compression function; i.e., find 48 columns — exactly 2 per block — which add up to 0.

Applying Wagner to FSB_{48}

Determine the number of lists for a Wagner attack on FSB_{48}.

- We choose 16 lists to solve this particular 48-sum problem. (16 is the highest power of 2 dividing 48).
- Each list entry will be the xor of three columns coming from one and a half blocks (no overlaps!)

Straightforward Wagner

- Applying Wagner's attack with 16 lists in a straightforward way means that we need to have at least $2^{\lceil 192/5 \rceil}$ entries per list.
- By clamping away 39 bits in each step we expect to get at least one collision after one run of the tree algorithm.

List entries

- For each list we generate more than twice the amount needed for a straightforward attack.
- Reduce amount of data by clamping away 2 bits $\Rightarrow 2^{38}$ entries per list (clamp 38 bits on each level)
- Ultimately we are not interested in the value of the entry; but in the column positions in the matrix that lead to this all-zero value.
  - Value-only representation
  - Positions-only representation: keep full positions; if we we need the value (or parts of it) it can be dynamically recomputed from the positions.
- Note: Unlike storage requirements for values the number of bytes for positions increases with increasing levels.

Storing positions

- Encode column positions of each entry in 40 bits (5 bytes) for the first level.
- The expected number of entries per list remains the same but the number of lists halves; so the total amount of data is the same on each level when using dynamic recomputation.
- Storing 16 lists with 2^{38} entries, each entry encoded in 5 bytes requires 20480 GB of storage space.
- The Coding and Cryptography Computer Cluster at Eindhoven University of Technology only has a total hard disk space of about 5440 GB, so we have to adapt our attack strategy to this limitation.

Adapt attack strategy

- Can handle at most $5 \cdot 2^{40}/2^4/5 = 2^{36}$ entries per list.

- A straightforward implementation would use lists of size 2^{36}: clamp 4 bits during list generation; this leads to 2^{36} values for each of the 16 lists.

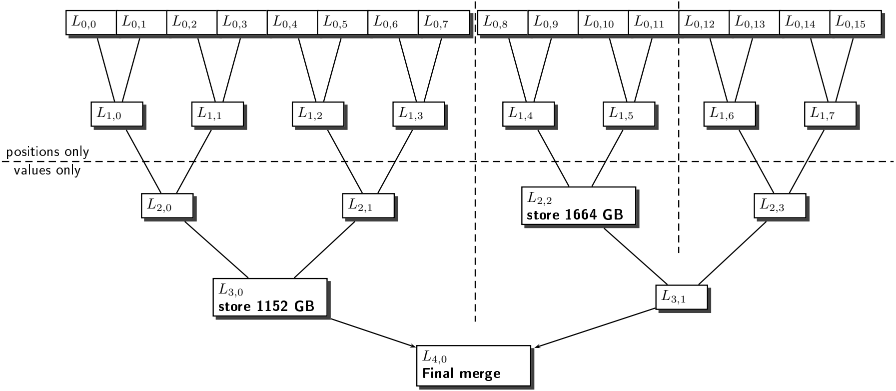
- We expect to run the attack 256.5 times until we find a collision.

Attack in two phases

Idea

- First phase: Figure out which clamping constants yield collision
- Second phase: Compute matrix positions yielding collision
- During phase one we do not have to store positions of entries
- On each level compress entries to shortest possible representation
  - Level 0: 5 bytes (positions only)
  - Level 1: 10 bytes (positions only)
  - Level 2: 13 bytes (values only)
  - Level 3: 9 bytes (values only)
- Use lists of size 2^{37}
- Clamp 3 bits through precomputation
- This leaves 4 bits "uncontrolled"

Attack Strategy



$$\Longrightarrow 1152 \text{ GB} + 1664 \text{ GB} + 2560 \text{ GB} = 5376 \text{ GB}$$

Our Strategy

- Continue the computation with different clamping contants until $L_{4,0}$ contains at least one entry
- Store the values in $L_{3,0}$ and $L_{3,1}$ that yield the collision
- Recompute $L_{3,0}$ and $L_{3,1}$ using positions-only representation to find positions in the matrix
- Expected:
  - $1\times$ Computation of $L_{3,0}$ (values only)
  - $1\times$ Computation of $L_{2,2}$ (values only)
  - $16.5\times$ Computation of $L_{2,3}$, $L_{3,1}$, $L_{4,0}$ (values only)
  - $1\times$ Computation of $L_{3,0}$ (positions only)
  - $1\times$ Computation of $L_{3,1}$ (positions only)

Finding the bottleneck(s)

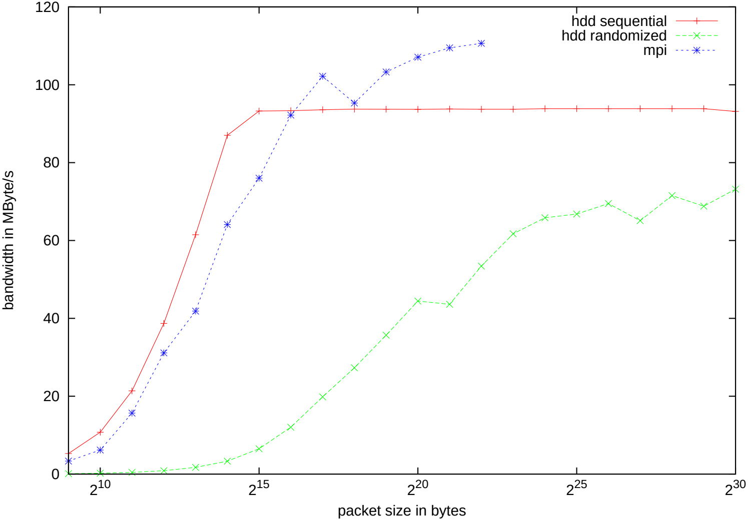
- Basically every byte needs to be stored, sent, and loaded.
- Possible performance bottlenecks
  - CPU computation power
  - Network throughput
  - Hard-disk throughput

Finding the bottleneck(s)

Technische Universiteit **Eindhoven** University of Technology

- Basically every byte needs to be stored, sent, and loaded.
- Possible performance bottlenecks
  - CPU computation power
  - Network throughput
  - Hard-disk throughput
- If the CPU is too slow we have to write faster code
- Determine network throughput: IBM MPI benchmark
- Determine hard-disk throughput: our own hard-disk benchmark
  - Direct I/O, no filesystem
  - Sequential and randomized access patterns

Finding the bottleneck(s)



Finding the bottleneck(s)

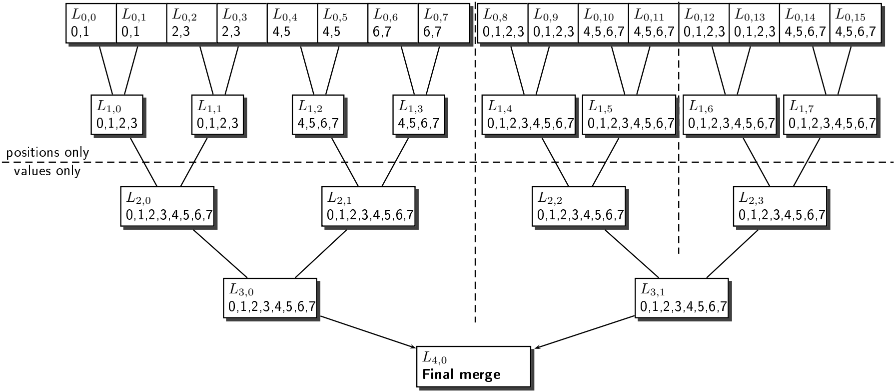
- Basically every byte needs to be stored, sent, and loaded.
- Possible performance bottlenecks
  - CPU computation power
  - Network throughput
  - Hard-disk throughput
- If the CPU is too slow we have to write faster code
- Determine network throughput: IBM MPI benchmark
- Determine hard-disk throughput: our own hard-disk benchmark
  - Direct I/O, no filesystem
  - Sequential and randomized access patterns

$\Longrightarrow$ Mainly bottlenecked by hard-disk throughput

Parallelization

- Distribute fractions of lists to nodes according to some of the bits relevant for sorting and merging on the next level
- Each node on each level holds two fractions of two lists
- Each node performs sort-and-merge on its list fractions

Parallelization



Parallelization

- Split fractions further into 512 parts of 640 MB each (presort, according to 9 bits)
- Sort and merge parts independently in memory
- Pipeline
  - Loading from hard disk into memory
  - Sorting of two parts
  - Merging of previously sorted parts
- Requires 6 parts in memory at the same time (3.75 GB)

Parallelization

- Split fractions further into 512 parts of 640 MB each (presort, according to 9 bits)
- Sort and merge parts independently in memory
- Pipeline
  - Loading from hard disk into memory
  - Sorting of two parts
  - Merging of previously sorted parts
- Requires 6 parts in memory at the same time (3.75 GB)
- Two blocks of operations:
  - Load, Sort, Merge, Send
  - Receive, Presort, Store

Parallelization

- Split fractions further into 512 parts of 640 MB each (presort, according to 9 bits)
- Sort and merge parts independently in memory
- Pipeline
  - Loading from hard disk into memory
  - Sorting of two parts
  - Merging of previously sorted parts
- Requires 6 parts in memory at the same time (3.75 GB)
- Two blocks of operations:
  - Load, Sort, Merge, Send
  - Receive, Presort, Store
- Observe: Bits known through node and presorting do not have to be stored in value-only representation
- That's how we get down to 13 and 9 bytes on levels 2 and 3 respectively

Ales instead of Files

- Each node uses a large data partition `/dev/sda1`
- Opened with `O_DIRECT` (without caching)
- Organize data in chunks of 1,198,080 Bytes ("ales")
- This value is a multiple of 9, 13, 40 (entry sizes) and 512 (for DMA)
- AleSystem also stores number of elements per part
- Throughput with sequential access (during list generation): $\sim$90 MB/sec
- Throughput with random access: $\sim$40 MB/sec

Timing Results

- Current benchmarks for phase 1:
  - Computation of list $L_{3,0}$: ~ 32 h (once)
  - Computation of list $L_{2,2}$: ~ 14 h (once)
  - Computation of list $L_{2,3}$: ~ 14 h (exp. 16.5×)
  - Computation of list $L_{3,1}$: ~ 4 h (exp. 16.5×)
  - Check for collision in $L_{3,0}$ and $L_{3,1}$: ~ 3.5 h (exp. 16.5×)
- Expected time for phase 1: $32 + 14 + 16.5 \cdot (14 + 4 + 3.5)$ = 400.7 h or 17 days
- Time for phase 2: ∼33 h per half-tree, in total ∼66 h
- Expected time in total: ∼19.5 days.

Timing Results

- Current benchmarks for phase 1:
  - Computation of list $L_{3,0}$: ~ 32 h (once)
  - Computation of list $L_{2,2}$: ~ 14 h (once)
  - Computation of list $L_{2,3}$: ~ 14 h (exp. 16.5×)
  - Computation of list $L_{3,1}$: ~ 4 h (exp. 16.5×)
  - Check for collision in $L_{3,0}$ and $L_{3,1}$: ~ 3.5 h (exp. 16.5×)
- Expected time for phase 1: $32 + 14 + 16.5 \cdot (14 + 4 + 3.5)$ = 400.7 h or 17 days
- Time for phase 2: ∼33 h per half-tree, in total ∼66 h
- Expected time in total: ∼19.5 days.
- Some parts of the code might be optimized further
- The attack is stateful so it is easy to exchange code with faster version

Further information

Paper: http://eprint.iacr.org/2009/292

Cluster: http://www.win.tue.nl/cccc/

Code: Will be available (public domain)