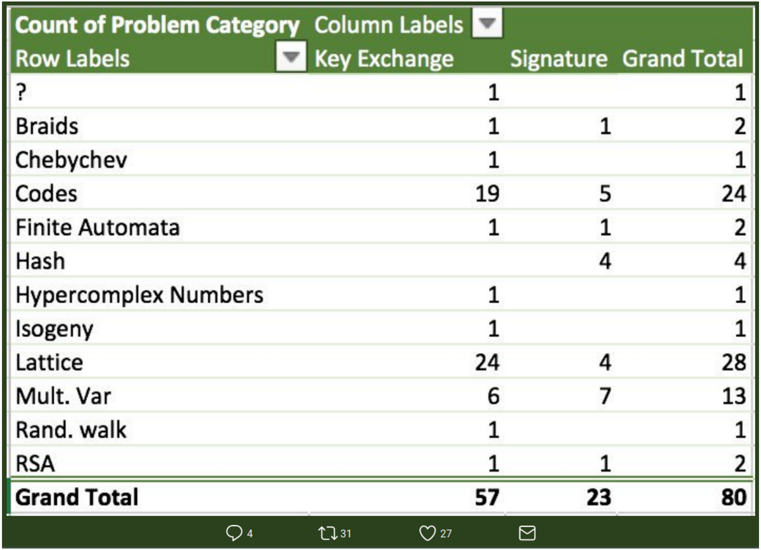
MAX PLANCK INSTITUTE
FOR SECURITY AND PRIVACY

An Introduction to Lattice-based KEMs

Peter Schwabe

December 17, 2020

The NIST competition



Count of Problem Category	Column Labels		
Row Labels	Key Exchange	Signature	Grand Total
?	1		1
Braids	1	1	2
Chebychev	1		1
Codes	19	5	24
Finite Automata	1	1	2
Hash		4	4
Hypercomplex Numbers	1		1
Isogeny	1		1
Lattice	24	4	28
Mult. Var	6	7	13
Rand. walk	1		1
RSA	1	1	2
Grand Total	**57**	**23**	**80**

Overview tweeted by Jacob Alperin-Sheriff on Dec 4, 2017.

The NIST competition: round 2

- Announcement planned at Real-World Crypto 2019

The NIST competition: round 2

- Announcement planned at Real-World Crypto 2019
- Due to US government lockdown slightly later

The NIST competition: round 2

- Announcement planned at Real-World Crypto 2019
- Due to US government lockdown slightly later

Encryption / Key agreement

- 9 lattice-based
- 7 code-based
- 1 isogeny-based

The NIST competition: round 2

- Announcement planned at Real-World Crypto 2019
- Due to US government lockdown slightly later

Encryption / Key agreement

- 9 lattice-based
- 7 code-based
- 1 isogeny-based

Signature schemes

- 3 lattice-based
- 2 symmetric-crypto based
- 4 MQ-based

The NIST competition: round 3

- Announcement planned for June 2020

The NIST competition: round 3

- Announcement planned for June 2020
- Due to pandemic (?) slightly later

The NIST competition: round 3

- Announcement planned for June 2020
- Due to pandemic (?) slightly later

Finalists

- 4 key-agreement schemes
  - 3 lattice-based
  - 1 code-based
- 3 signature schemes
  - 2 lattice-based
  - 1 MQ-based

The NIST competition: round 3

- Announcement planned for June 2020
- Due to pandemic (?) slightly later

Finalists

- 4 key-agreement schemes
    - 3 lattice-based
    - 1 code-based
- 3 signature schemes
    - 2 lattice-based
    - 1 MQ-based

Alternate schemes

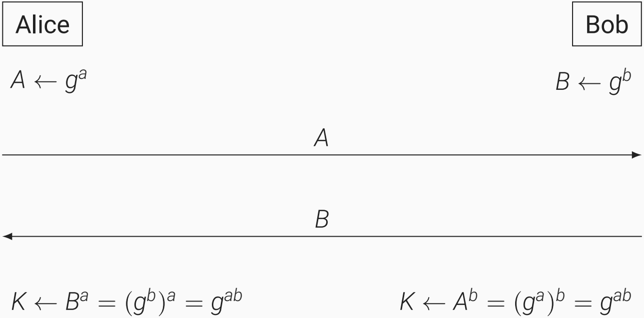
- 5 key-agreement schemes
    - 2 lattice-based
    - 2 code-based
    - 1 isogeny-based
- 3 signature schemes
    - 2 symmetric-crypto based
    - 1 MQ-based

What NIST means by “Key exchange”

Key encapsulation mechanisms (KEMs)

- $(\mathrm{pk}, \mathrm{sk}) \leftarrow \mathrm{KeyGen}()$
- $(c, k) \leftarrow \mathrm{Encaps}(\mathrm{pk})$
- $k \leftarrow \mathrm{Decaps}(c, \mathrm{sk})$

A reminder of Diffie-Hellman



Alice

Bob

$A \leftarrow g^a$

$B \leftarrow g^b$

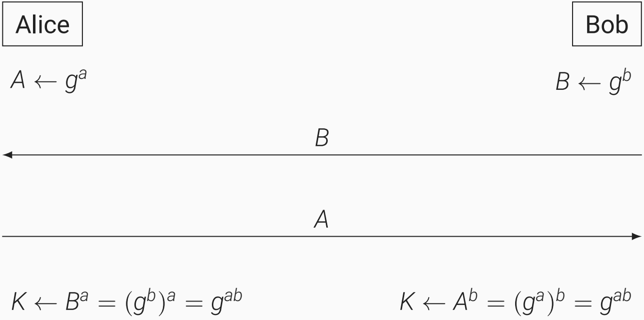
A

B

$K \leftarrow B^a = (g^b)^a = g^{ab}$

$K \leftarrow A^b = (g^a)^b = g^{ab}$

A reminder of Diffie-Hellman



Alice

Bob

$A \leftarrow g^a$

$B \leftarrow g^b$

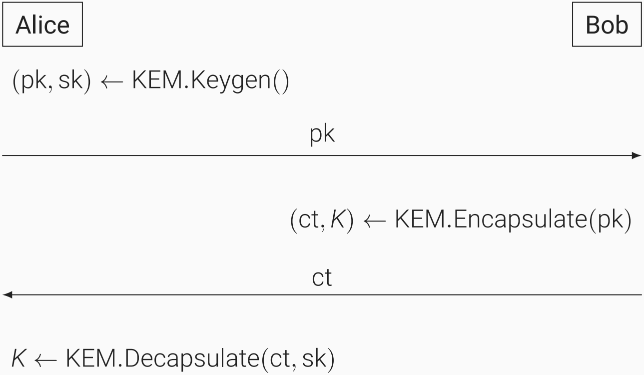
$\xleftarrow{\quad B \quad}$

$\xrightarrow{\quad A \quad}$

$K \leftarrow B^a = (g^b)^a = g^{ab}$

$K \leftarrow A^b = (g^a)^b = g^{ab}$

KEMs: as close as you’ll get to DH



Alice

Bob

$(\mathrm{pk}, \mathrm{sk}) \leftarrow \mathrm{KEM.Keygen}()$

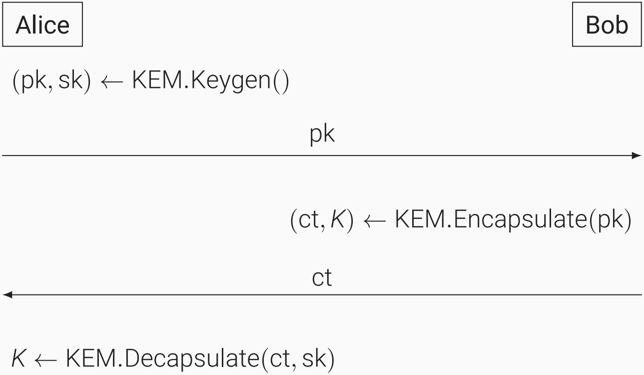
pk

$(\mathrm{ct}, K) \leftarrow \mathrm{KEM.Encapsulate}(\mathrm{pk})$

ct

$K \leftarrow \mathrm{KEM.Decapsulate}(\mathrm{ct}, \mathrm{sk})$

KEMs: as close as you’ll get to DH*



Alice

Bob

$(\mathrm{pk}, \mathrm{sk}) \leftarrow \mathrm{KEM.Keygen}()$

$\xrightarrow{\mathrm{pk}}$

$(\mathrm{ct}, K) \leftarrow \mathrm{KEM.Encapsulate}(\mathrm{pk})$

$\xleftarrow{\mathrm{ct}}$

$K \leftarrow \mathrm{KEM.Decapsulate}(\mathrm{ct}, \mathrm{sk})$

*Except with CSIDH (Castryck, Lange, Martindale, Renes, Panny, 2018)

Lattice-based KEMs

Google Security Blog

The latest news and insights from Google on security and safety on the Internet

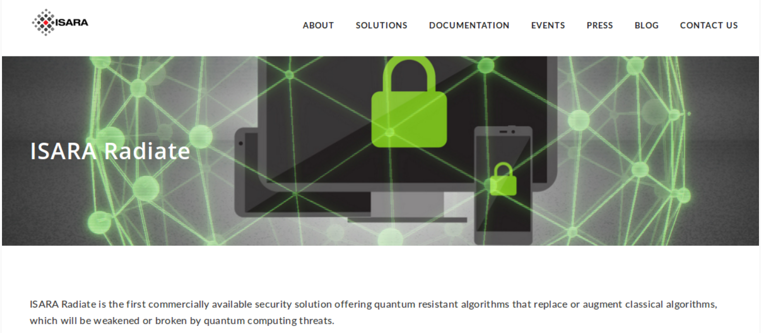
Experimenting with Post-Quantum Cryptography

July 7, 2016

Posted by Matt Braithwaite, Software Engineer

“We’re indebted to Erdem Alkim, Léo Ducas, Thomas Pöppelmann and Peter Schwabe, the researchers who developed “New Hope”, the post-quantum algorithm that we selected for this experiment.”

`https://security.googleblog.com/2016/07/experimenting-with-post-quantum.html`

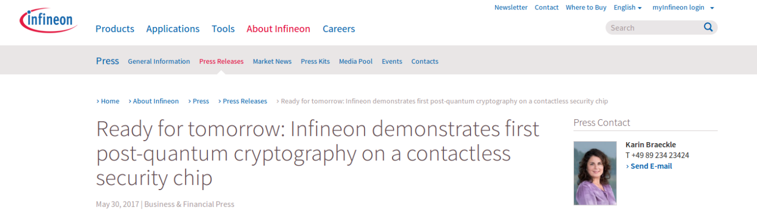


ISARA Radiate

ISARA Radiate is the first commercially available security solution offering quantum resistant algorithms that replace or augment classical algorithms, which will be weakened or broken by quantum computing threats.

“Key Agreement using the ‘NewHope’ lattice-based algorithm detailed in the New Hope paper, and LUKE (Lattice-based Unique Key Exchange), an ISARA speed-optimized version of the NewHope algorithm.”

`https://www.isara.com/isara-radiate/`



Ready for tomorrow: Infineon demonstrates first post-quantum cryptography on a contactless security chip

May 30, 2017 | Business & Financial Press

Press Contact

Karin Braeckle
T +49 89 234 23424
› **Send E-mail**

“The deployed algorithm is a variant of “New Hope”, a quantum-resistant cryptosystem”

`https://www.infineon.com/cms/en/about-infineon/press/press-releases/2017/INFCCS201705-056.html`

Learning with errors (LWE)

- Given uniform $\mathbf{A} \in \mathbb{Z}_q^{k \times \ell}$
- Given “noise distribution” χ
- Given samples $\mathbf{As} + \mathbf{e}$, with $\mathbf{e} \leftarrow \chi$

Learning with errors (LWE)

- Given uniform $\mathbf{A} \in \mathbb{Z}_q^{k\times\ell}$
- Given “noise distribution” χ
- Given samples $\mathbf{A}\mathbf{s} + \mathbf{e}$, with $\mathbf{e}\leftarrow\chi$
- Search version: find $\mathbf{s}$
- Decision version: distinguish from uniform random

Learning with rounding (LWR)

- Given uniform $\mathbf{A} \in \mathbb{Z}_q^{k \times \ell}$
- Given samples $\lceil \mathbf{As} \rfloor_p$, with $p < q$

Learning with rounding (LWR)

- Given uniform $\mathbf{A} \in \mathbb{Z}_q^{k \times \ell}$
- Given samples $\lceil \mathbf{A}\mathbf{s} \rfloor_p$, with $p < q$
- Search version: find $\mathbf{s}$
- Decision version: distinguish from uniform random

Using structured lattices

- Problem with LWE-based cryptosystems: public-key size
- Only NIST candidate exclusively using standard LWE: FrodoKEM

Using structured lattices

- Problem with LWE-based cryptosystems: public-key size
- Only NIST candidate exclusively using standard LWE: FrodoKEM
- Idea to solve this: allow structured matrix $\mathbf{A}$, e.g.,

Using structured lattices

- Problem with LWE-based cryptosystems: public-key size
- Only NIST candidate exclusively using standard LWE: FrodoKEM
- Idea to solve this: allow structured matrix $\mathbf{A}$, e.g.,
    - NewHope: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$; n a power of 2, q prime

Using structured lattices

- Problem with LWE-based cryptosystems: public-key size
- Only NIST candidate exclusively using standard LWE: FrodoKEM
- Idea to solve this: allow structured matrix $\mathbf{A}$, e.g.,
  - NewHope: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$; n a power of 2, q prime
  - NTRU: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n - 1)$; n prime, q a power of 2

Using structured lattices

- Problem with LWE-based cryptosystems: public-key size
- Only NIST candidate exclusively using standard LWE: FrodoKEM
- Idea to solve this: allow structured matrix $\mathbf{A}$, e.g.,
    - NewHope: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$; n a power of 2, q prime
    - NTRU: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n - 1)$; n prime, q a power of 2
    - NTRU Prime: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n - X - 1)$; q prime, n prime

Using structured lattices

- Problem with LWE-based cryptosystems: public-key size
- Only NIST candidate exclusively using standard LWE: FrodoKEM
- Idea to solve this: allow structured matrix $\mathbf{A}$, e.g.,
    - NewHope: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$; n a power of 2, q prime
    - NTRU: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n - 1)$; n prime, q a power of 2
    - NTRU Prime: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n - X - 1)$; q prime, n prime
    - Kyber/Saber: use small-dimension matrices and vectors over $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^{256} + 1)$

Using structured lattices

- Problem with LWE-based cryptosystems: public-key size
- Only NIST candidate exclusively using standard LWE: FrodoKEM
- Idea to solve this: allow structured matrix $\mathbf{A}$, e.g.,
    - NewHope: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$; n a power of 2, q prime
    - NTRU: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n - 1)$; n prime, q a power of 2
    - NTRU Prime: work in $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n - X - 1)$; q prime, n prime
    - Kyber/Saber: use small-dimension matrices and vectors over $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^{256} + 1)$
- Perform arithmetic on (vectors of) polynomials instead of vectors/matrices over $\mathbb{Z}_q$

How to build a KEM?

Alice (server)		Bob (client)
$\mathbf{s}, \mathbf{e} \overset{\$}{\leftarrow} \chi$		$\mathbf{s}', \mathbf{e}' \overset{\$}{\leftarrow} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{\mathbf{b}}$	$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
	$\xleftarrow{\mathbf{u}}$	

Alice has $\mathbf{v} = \mathbf{us} = \mathbf{ass}' + \mathbf{e}'\mathbf{s}$

Bob has $\mathbf{v}' = \mathbf{bs}' = \mathbf{ass}' + \mathbf{es}'$

- Secret and noise polynomials $\mathbf{s}, \mathbf{s}', \mathbf{e}, \mathbf{e}'$ are small
- $\mathbf{v}$ and $\mathbf{v}'$ are *approximately* the same

How to build a KEM, part 2

Alice		Bob
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}' \quad \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{(\mathbf{b} \qquad)}$	
		$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{b}\mathbf{s}'$
$\mathbf{v}' \leftarrow \mathbf{u}\mathbf{s}$	$\xleftarrow{(\mathbf{u} \quad)}$	

How to build a KEM, part 2

Alice		Bob
$\mathit{seed} \overset{\$}{\leftarrow} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(\mathit{seed}))$		
$\mathbf{s}, \mathbf{e} \overset{\$}{\leftarrow} \chi$		$\mathbf{s}', \mathbf{e}' \overset{\$}{\leftarrow} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, \mathit{seed})}$	$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(\mathit{seed}))$
		$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{b}\mathbf{s}'$
$\mathbf{v}' \leftarrow \mathbf{u}\mathbf{s}$	$\xleftarrow{(\mathbf{u} \quad)}$	

How to build a KEM, part 2

Alice		Bob
$\mathit{seed} \overset{\$}{\leftarrow} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \mathrm{Parse}(\mathrm{XOF}(\mathit{seed}))$		
$\mathbf{s}, \mathbf{e} \overset{\$}{\leftarrow} \chi$		$\mathbf{s}', \mathbf{e}' \overset{\$}{\leftarrow} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, \mathit{seed})}$	$\mathbf{a} \leftarrow \mathrm{Parse}(\mathrm{XOF}(\mathit{seed}))$
		$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{b}\mathbf{s}'$
		$k \overset{\$}{\leftarrow} \{0,1\}^{n}$
		$\mathbf{k} \leftarrow \mathrm{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{u}\mathbf{s}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$

How to build a KEM, part 2

Alice		Bob
$seed \xleftarrow{\$} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(seed))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, seed)}$	$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(seed))$
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}' + \mathbf{e}''$
		$k \xleftarrow{\$} \{0,1\}^{n}$
		$\mathbf{k} \leftarrow \mathsf{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$

How to build a KEM, part 2

Alice		Bob
$seed \xleftarrow{\$} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \text{Parse}(\text{XOF}(seed))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, seed)}$	$\mathbf{a} \leftarrow \text{Parse}(\text{XOF}(seed))$
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}' + \mathbf{e}''$
		$k \xleftarrow{\$} \{0,1\}^n$
		$\mathbf{k} \leftarrow \text{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$
$\mathbf{k}' \leftarrow \mathbf{c} - \mathbf{v}'$		

How to build a KEM, part 2

Alice		Bob
$\mathit{seed} \overset{\$}{\leftarrow} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(\mathit{seed}))$		
$\mathbf{s}, \mathbf{e} \overset{\$}{\leftarrow} \chi$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \overset{\$}{\leftarrow} \chi$
$\mathbf{b} \leftarrow \mathbf{a}\mathbf{s} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, \mathit{seed})}$	$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(\mathit{seed}))$
		$\mathbf{u} \leftarrow \mathbf{a}\mathbf{s}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{b}\mathbf{s}' + \mathbf{e}''$
		$k \overset{\$}{\leftarrow} \{0,1\}^{n}$
		$\mathbf{k} \leftarrow \mathsf{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{u}\mathbf{s}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$
$\mathbf{k}' \leftarrow \mathbf{c} - \mathbf{v}'$		$\mu \leftarrow \mathsf{Extract}(\mathbf{k})$
$\mu \leftarrow \mathsf{Extract}(\mathbf{k}')$		

How to build a KEM, part 2

Alice		Bob
$\mathit{seed} \xleftarrow{\$} \{0,1\}^{256}$		
$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(\mathit{seed}))$		
$\mathbf{s}, \mathbf{e} \xleftarrow{\$} \chi$		$\mathbf{s}', \mathbf{e}', \mathbf{e}'' \xleftarrow{\$} \chi$
$\mathbf{b} \leftarrow \mathbf{as} + \mathbf{e}$	$\xrightarrow{(\mathbf{b}, \mathit{seed})}$	$\mathbf{a} \leftarrow \mathsf{Parse}(\mathsf{XOF}(\mathit{seed}))$
		$\mathbf{u} \leftarrow \mathbf{as}' + \mathbf{e}'$
		$\mathbf{v} \leftarrow \mathbf{bs}' + \mathbf{e}''$
		$k \xleftarrow{\$} \{0,1\}^n$
		$\mathbf{k} \leftarrow \mathsf{Encode}(k)$
$\mathbf{v}' \leftarrow \mathbf{us}$	$\xleftarrow{(\mathbf{u}, \mathbf{c})}$	$\mathbf{c} \leftarrow \mathbf{v} + \mathbf{k}$
$\mathbf{k}' \leftarrow \mathbf{c} - \mathbf{v}'$		$\mu \leftarrow \mathsf{Extract}(\mathbf{k})$
$\mu \leftarrow \mathsf{Extract}(\mathbf{k}')$		

This is LPR encryption, written as KEM (except for generation of $\mathbf{a}$)

Encode and Extract

- Encoding in LPR encryption: map *n* bits to *n* coefficients:
    - A zero bit maps to 0
    - A one bit maps to $q/2$
- Idea: Noise affects low bits of coefficients, put data into high bits

Encode and Extract

- Encoding in LPR encryption: map *n* bits to *n* coefficients:
    - A zero bit maps to 0
    - A one bit maps to $q/2$
- Idea: Noise affects low bits of coefficients, put data into high bits
- Decode: map coefficient into $[-q/2, q/2]$
    - Closer to 0 (i.e., in $[-q/4, q/4]$): set bit to zero
    - Closer to $\pm q/2$: set bit to one

From passive to CCA security

- The base scheme does not have active security
- Attacker can choose arbitrary noise, learns $\mathbf{s}$ from failures

From passive to CCA security

- The base scheme does not have active security
- Attacker can choose arbitrary noise, learns $\mathbf{s}$ from failures
- Fujisaki-Okamoto transform (sketched):

Alice (Server)		Bob (Client)
Gen():		
pk, sk$\leftarrow$KeyGen()		Enc(seed, $\mathbf{b}$):
seed, $\mathbf{b}\leftarrow$pk	$\xrightarrow{\text{seed},\mathbf{b}}$	$x\leftarrow\{0,\dots,255\}^{32}$ k, coins$\leftarrow$SHA3-512(x)
	$\xleftarrow{\mathbf{u},v}$	$\mathbf{u}, v\leftarrow$Encrypt((seed, $\mathbf{b}$), x, coins)
Dec($\mathbf{s}$, ($\mathbf{u}$, v)): $x'\leftarrow$ Decrypt($\mathbf{s}$, ($\mathbf{u}$, v)) k', $coins'\leftarrow$SHA3-512(x') $\mathbf{u}'$, $v'\leftarrow$Encrypt((seed, $\mathbf{b}$), x', coins$'$) verify if $(\mathbf{u}', v') = (\mathbf{u}, v)$		

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- Keygen:
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- Keygen:
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
- Encrypt:
  - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
  - Sample random small-coefficient polynomial $\mathbf{r} \in \mathcal{R}_q$
  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- Keygen:
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
- Encrypt:
  - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
  - Sample random small-coefficient polynomial $\mathbf{r} \in \mathcal{R}_q$
  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$
- Decrypt:
  - Compute $\mathbf{v} = \mathbf{f} \cdot \mathbf{e}$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- Keygen:
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
- Encrypt:
  - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
  - Sample random small-coefficient polynomial $\mathbf{r} \in \mathcal{R}_q$
  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$
- Decrypt:
  - Compute $\mathbf{v} = \mathbf{f} \cdot \mathbf{e} = \mathbf{f} \cdot (\mathbf{r} \cdot \mathbf{h} + \mathbf{m})$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- Keygen:
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
- Encrypt:
  - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
  - Sample random small-coefficient polynomial $\mathbf{r} \in \mathcal{R}_q$
  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$
- Decrypt:
  - Compute $\mathbf{v} = \mathbf{f} \cdot \mathbf{e} = \mathbf{f} \cdot (\mathbf{r} \cdot \mathbf{h} + \mathbf{m}) = \mathbf{f}(\mathbf{r} \cdot (p\mathbf{f}_q\mathbf{g}) + \mathbf{m})$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- Keygen:
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
- Encrypt:
  - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
  - Sample random small-coefficient polynomial $\mathbf{r} \in \mathcal{R}_q$
  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$
- Decrypt:
  - Compute $\mathbf{v} = \mathbf{f} \cdot \mathbf{e} = \mathbf{f} \cdot (\mathbf{r} \cdot \mathbf{h} + \mathbf{m}) = \mathbf{f}(\mathbf{r} \cdot (p\mathbf{f}_q\mathbf{g}) + \mathbf{m}) = p\mathbf{r}\mathbf{g} + \mathbf{f} \cdot \mathbf{m}$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- Keygen:
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
- Encrypt:
  - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
  - Sample random small-coefficient polynomial $\mathbf{r} \in \mathcal{R}_q$
  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$
- Decrypt:
  - Compute $\mathbf{v} = \mathbf{f} \cdot \mathbf{e} = \mathbf{f} \cdot (\mathbf{r} \cdot \mathbf{h} + \mathbf{m}) = \mathbf{f}(\mathbf{r} \cdot (p\mathbf{f}_q\mathbf{g}) + \mathbf{m}) = p\mathbf{r}\mathbf{g} + \mathbf{f} \cdot \mathbf{m}$
  - Compute $\mathbf{m} = \mathbf{v} \cdot \mathbf{f}_p \mod p$

Design space 0: The NTRU approach

- Historically first: NTRU
- Use parameters q and $p = 3$
- **Keygen:**
  - Find $\mathbf{f}, \mathbf{g} \in \mathcal{R}_q$ and $\mathbf{f}_q = \mathbf{f}^{-1} \mod q, \mathbf{f}_p = \mathbf{f}^{-1} \mod p$
  - public key: $\mathbf{h} = p\mathbf{f}_q\mathbf{g}$, secret key: $(\mathbf{f}, \mathbf{f}_p)$
- **Encrypt:**
  - Map message m to $\mathbf{m} \in \mathcal{R}_q$ with coefficients in $\{-1, 0, 1\}$
  - Sample random small-coefficient polynomial $\mathbf{r} \in \mathcal{R}_q$
  - Compute ciphertext $\mathbf{e} = \mathbf{r} \cdot \mathbf{h} + \mathbf{m}$
- **Decrypt:**
  - Compute $\mathbf{v} = \mathbf{f} \cdot \mathbf{e} = \mathbf{f} \cdot (\mathbf{r} \cdot \mathbf{h} + \mathbf{m}) = \mathbf{f}(\mathbf{r} \cdot (p\mathbf{f}_q\mathbf{g}) + \mathbf{m}) = p\mathbf{r}\mathbf{g} + \mathbf{f} \cdot \mathbf{m}$
  - Compute $\mathbf{m} = \mathbf{v} \cdot \mathbf{f}_p \mod p$
- Advantages/Disadvantages compared to LPR:
  - Asymptotically weaker than Ring-LWE approach
  - Slower keygen, but faster encryption/decryption

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
  - q typically either prime or a power of two
  - f typically of degree between 512 and 1024

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
  - q typically either prime or a power of two
  - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime   (NTRU)

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
    - q typically either prime or a power of two
    - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime (NTRU)
- **Second option:** $q = 2^k$, $f = (X^n + 1)$, $n = 2^m$ (Saber)

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
    - q typically either prime or a power of two
    - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime (NTRU)
- **Second option:** $q = 2^k$, $f = (X^n + 1)$, $n = 2^m$ (Saber)
- **Third option:** $q = 2^k$, $f = \Phi_{n+1}$, $n + 1$ prime (Round5)

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
    - q typically either prime or a power of two
    - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime (NTRU)
- **Second option:** $q = 2^k$, $f = (X^n + 1)$, $n = 2^m$ (Saber)
- **Third option:** $q = 2^k$, $f = \Phi_{n+1}$, $n + 1$ prime (Round5)
- **Fourth option:** q prime, $f = (X^n + 1) = \Phi_{2n}$, $n = 2^m$ (NewHope, Kyber, LAC)

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
    - q typically either prime or a power of two
    - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime (NTRU)
- **Second option:** $q = 2^k$, $f = (X^n + 1)$, $n = 2^m$ (Saber)
- **Third option:** $q = 2^k$, $f = \Phi_{n+1}$, $n + 1$ prime (Round5)
- **Fourth option:** q prime, $f = (X^n + 1) = \Phi_{2n}$, $n = 2^m$ (NewHope, Kyber, LAC)
- **Fifth option:** q prime, $f = (X^n - X - 1)$ irreducible, n prime (NTRU Prime)

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
  - q typically either prime or a power of two
  - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime (NTRU)
- **Second option:** $q = 2^k$, $f = (X^n + 1)$, $n = 2^m$ (Saber)
- **Third option:** $q = 2^k$, $f = \Phi_{n+1}$, $n + 1$ prime (Round5)
- **Fourth option:** q prime, $f = (X^n + 1) = \Phi_{2n}$, $n = 2^m$ (NewHope, Kyber, LAC)
- **Fifth option:** q prime, $f = (X^n - X - 1)$ irreducible, n prime (NTRU Prime)
- **Sixth option:** ThreeBears works on large integers instead of polynomials

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
  - q typically either prime or a power of two
  - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime (NTRU)
- **Second option:** $q = 2^k$, $f = (X^n + 1)$, $n = 2^m$ (Saber)
- **Third option:** $q = 2^k$, $f = \Phi_{n+1}$, $n + 1$ prime (Round5)
- **Fourth option:** q prime, $f = (X^n + 1) = \Phi_{2n}$, $n = 2^m$ (NewHope, Kyber, LAC)
- **Fifth option:** q prime, $f = (X^n - X - 1)$ irreducible, n prime (NTRU Prime)
- **Sixth option:** ThreeBears works on large integers instead of polynomials
- No proof that any option is more or less secure

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
  - q typically either prime or a power of two
  - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime (NTRU)
- **Second option:** $q = 2^k$, $f = (X^n + 1)$, $n = 2^m$ (Saber)
- **Third option:** $q = 2^k$, $f = \Phi_{n+1}$, $n + 1$ prime (Round5)
- **Fourth option:** q prime, $f = (X^n + 1) = \Phi_{2n}$, $n = 2^m$ (NewHope, Kyber, LAC)
- **Fifth option:** q prime, $f = (X^n - X - 1)$ irreducible, n prime (NTRU Prime)
- **Sixth option:** ThreeBears works on large integers instead of polynomials
- No proof that any option is more or less secure
- NTRU Prime advertises "less structure" in their $\mathcal{R}_q$

Design space 1: What ring?

- Structured lattice-based schemes use ring $\mathcal{R}_q = \mathbb{Z}_q[X]/f$
    - q typically either prime or a power of two
    - f typically of degree between 512 and 1024
- **First option:** $q = 2^k$, $f = (X^n - 1)$, n prime (NTRU)
- **Second option:** $q = 2^k$, $f = (X^n + 1)$, $n = 2^m$ (Saber)
- **Third option:** $q = 2^k$, $f = \Phi_{n+1}$, $n + 1$ prime (Round5)
- **Fourth option:** q prime, $f = (X^n + 1) = \Phi_{2n}$, $n = 2^m$ (NewHope, Kyber, LAC)
- **Fifth option:** q prime, $f = (X^n - X - 1)$ irreducible, n prime (NTRU Prime)
- **Sixth option:** ThreeBears works on large integers instead of polynomials
- No proof that any option is more or less secure
- NTRU Prime advertises “less structure” in their $\mathcal{R}_q$
- NewHope and Kyber have fastest (NTT-based) arithmetic

Design space 2: module vs. ring?

- "Traditionally", work directly with elements of $\mathcal{R}_q$ ("Ring-LWE")
- Alternative: Module-LWE (MLWE):
  - Choose smaller n, e.g., $n = 256$ (Kyber, Saber, ThreeBears)
  - Work with small-dimension matrices and vectors over $\mathcal{R}_q$

Design space 2: module vs. ring?

- “Traditionally”, work directly with elements of $\mathcal{R}_q$ (“Ring-LWE”)
- Alternative: Module-LWE (MLWE):
  - Choose smaller *n*, e.g., $n = 256$ (Kyber, Saber, ThreeBears)
  - Work with small-dimension matrices and vectors over $\mathcal{R}_q$
- MLWE encrypts shorter messages than Ring-LWE

Design space 2: module vs. ring?

- “Traditionally”, work directly with elements of $\mathcal{R}_q$ (“Ring-LWE”)
- Alternative: Module-LWE (MLWE):
  - Choose smaller *n*, e.g., $n = 256$ (Kyber, Saber, ThreeBears)
  - Work with small-dimension matrices and vectors over $\mathcal{R}_q$
- MLWE encrypts shorter messages than Ring-LWE
- MLWE eliminates some of the structure of Ring-LWE

Design space 2: module vs. ring?

- “Traditionally”, work directly with elements of $\mathcal{R}_q$ (“Ring-LWE”)
- Alternative: Module-LWE (MLWE):
    - Choose smaller n, e.g., $n = 256$ (Kyber, Saber, ThreeBears)
    - Work with small-dimension matrices and vectors over $\mathcal{R}_q$
- MLWE encrypts shorter messages than Ring-LWE
- MLWE eliminates some of the structure of Ring-LWE
- MLWE can very easily scale security (change dimension of matrix):
    - Optimize arithmetic in $\mathcal{R}_q$ once
    - Use same optimized $\mathcal{R}_q$ arithmetic for all security levels

Design space 3: what noise?

- Need to sample noise (for LWE schemes) and small secrets
- More noise means
  - more security from the underlying hard problem
  - higher failure probability of decryption

Design space 3: what noise?

- Need to sample noise (for LWE schemes) and small secrets
- More noise means
  - more security from the underlying hard problem
  - higher failure probability of decryption
- Three main choices to make:
  - Narrow or wide noise
    - Narrow noise (e.g., in $\{-1, 0, 1\}$) not conservative
    - Wide noise requires larger q (or more failures)
    - Larger q means larger public key and ciphertext

Design space 3: what noise?

- Need to sample noise (for LWE schemes) and small secrets
- More noise means
  - more security from the underlying hard problem
  - higher failure probability of decryption
- Three main choices to make:
  - **Narrow or wide noise**
    - Narrow noise (e.g., in $\{-1, 0, 1\}$) not conservative
    - Wide noise requires larger q (or more failures)
    - Larger q means larger public key and ciphertext
  - **LWE or LWR**
    - LWE considered more conservative (independent noise)
    - LWR easier to implement (no noise sampling)
    - LWR allows more compact public key and ciphertext

Design space 3: what noise?

- Need to sample noise (for LWE schemes) and small secrets
- More noise means
    - more security from the underlying hard problem
    - higher failure probability of decryption
- Three main choices to make:
    - Narrow or wide noise
        - Narrow noise (e.g., in $\{-1, 0, 1\}$) not conservative
        - Wide noise requires larger q (or more failures)
        - Larger q means larger public key and ciphertext
    - LWE or LWR
        - LWE considered more conservative (independent noise)
        - LWR easier to implement (no noise sampling)
        - LWR allows more compact public key and ciphertext
    - Fixed-weight noise or not?
        - Fixed-weight noise needs random permutation (sorting)
        - Naive implementations leak secrets through timing
        - Advantage of fixed-weight: easier to bound (or eliminate) decryption failures

Design space 4: allow failures?

- Can avoid decryption failures entirely (NTRU, NTRU Prime)
- Advantage:
    - Easier CCA security transform and analysis
- Disadvantage:
    - Need to limit noise (or have larger q)

Design space 4: allow failures?

- Can avoid decryption failures entirely (NTRU, NTRU Prime)
- Advantage:
  - Easier CCA security transform and analysis
- Disadvantage:
  - Need to limit noise (or have larger q)
- For passive-security-only can go the other way:
  - Allow failure probability of, e.g., 2^{-30}
  - Reduce size of public key and ciphertext

Design space 4: allow failures?

- Can avoid decryption failures entirely (NTRU, NTRU Prime)
- Advantage:
  - Easier CCA security transform and analysis
- Disadvantage:
  - Need to limit noise (or have larger q)
- For passive-security-only can go the other way:
  - Allow failure probability of, e.g., 2^{-30}
  - Reduce size of public key and ciphertext
- Active (CCA) security needs negligible failure probability

Design space 5: public parameters?

- “Traditional” approach to choosing $\mathbf{a}$ in LWE/LWR schemes:

  “Let $\mathbf{a}$ *be a uniformly random…”*

Design space 5: public parameters?

- "Traditional" approach to choosing $\mathbf{a}$ in LWE/LWR schemes:

  "Let $\mathbf{a}$ *be a uniformly random. . . "*

- Before NewHope: *real-world* approach: generate fixed $\mathbf{a}$ once

Design space 5: public parameters?

- "Traditional" approach to choosing $\mathbf{a}$ in LWE/LWR schemes:

  "Let $\mathbf{a}$ be a uniformly random…"

- Before NewHope: *real-world* approach: generate fixed $\mathbf{a}$ once
- What if $\mathbf{a}$ is backdoored?
- Parameter-generating authority can break key exchange
- "Solution": Nothing-up-my-sleeves (involves endless discussion!)

Design space 5: public parameters?

- “Traditional” approach to choosing $\mathbf{a}$ in LWE/LWR schemes:

  “Let $\mathbf{a}$ *be a uniformly random. . . ”*
- Before NewHope: *real-world* approach: generate fixed $\mathbf{a}$ once
- What if $\mathbf{a}$ is backdoored?
- Parameter-generating authority can break key exchange
- “Solution”: Nothing-up-my-sleeves (involves endless discussion!)
- Even without backdoor:
  - Perform massive precomputation based on $\mathbf{a}$
  - Use precomputation to break *all* key exchanges
  - Infeasible today, but who knows. . .
  - Attack in the spirit of Logjam

Design space 5: public parameters?

- "Traditional" approach to choosing $\mathbf{a}$ in LWE/LWR schemes:

  "Let $\mathbf{a}$ *be a uniformly random…"*

- Before NewHope: *real-world* approach: generate fixed $\mathbf{a}$ once
- What if $\mathbf{a}$ is backdoored?
- Parameter-generating authority can break key exchange
- "Solution": Nothing-up-my-sleeves (involves endless discussion!)
- Even without backdoor:
  - Perform massive precomputation based on $\mathbf{a}$
  - Use precomputation to break *all* key exchanges
  - Infeasible today, but who knows…
  - Attack in the spirit of Logjam
- Solution in NewHope: Choose a fresh $\mathbf{a}$ every time
- Server can cache $\mathbf{a}$ for some time (e.g., 1h)
- All NIST PQC candidates now use this approach

Design space 6: error-correcting codes?

- Ring-LWE/LWR schemes work with polynomials of > 256 coefficients
- “Encrypt” messages of > 256 bits
- **Need to encrypt** only 256-bit key
- Question: How do we put those additional bits to use?
- Answer: Use error-correcting code (ECC) to reduce failure probability

Design space 6: error-correcting codes?

- Ring-LWE/LWR schemes work with polynomials of > 256 coefficients
- "Encrypt" messages of > 256 bits
- **Need to encrypt** only 256-bit key
- Question: How do we put those additional bits to use?
- Answer: Use error-correcting code (ECC) to reduce failure probability
- NewHope: very simple threshold decoding

Design space 6: error-correcting codes?

- Ring-LWE/LWR schemes work with polynomials of > 256 coefficients
- "Encrypt" messages of > 256 bits
- **Need to encrypt** only 256-bit key
- Question: How do we put those additional bits to use?
- Answer: Use error-correcting code (ECC) to reduce failure probability
- NewHope: very simple threshold decoding
- LAC, Round5: more advanced ECC
  - Correct more errors, obtain smaller public key and ciphertext
  - More complex to implement, in particular without leaking through timing

Design space 7: CCA security?

- Ephemeral key exchange does not need CCA security
- Can offer passively secure version
- Protocols will combine this with signatures for authentication

Design space 7: CCA security?

- Ephemeral key exchange does not need CCA security
- Can offer passively secure version
- Protocols will combine this with signatures for authentication
- Advantages:
    - Higher failure probability → more compact
    - Simpler to implement, no CCA transform
    - More flexibility for secret/noise generation

Design space 7: CCA security?

- Ephemeral key exchange does not need CCA security
- Can offer passively secure version
- Protocols will combine this with signatures for authentication
- Advantages:
    - Higher failure probability → more compact
    - Simpler to implement, no CCA transform
    - More flexibility for secret/noise generation
- Disadvantages:
    - Less robust (will somebody reuse keys?)
    - More options (CCA vs. CPA): easier to make mistakes

Design space 8: CCA transforms

- General Fujisaki-Okamoto principle is the same for most KEMs (exception: NTRU)
- Tweaks to FO transform:
  - Hash public-key into coins: multitarget protection (for non-zero failure probability)

Design space 8: CCA transforms

- General Fujisaki-Okamoto principle is the same for most KEMs (exception: NTRU)
- Tweaks to FO transform:
  - Hash public-key into coins: multitarget protection (for non-zero failure probability)
  - Hash public-key into shared key: KEM becomes contributory

Design space 8: CCA transforms

- General Fujisaki-Okamoto principle is the same for most KEMs (exception: NTRU)
- Tweaks to FO transform:
  - Hash public-key into coins: multitarget protection (for non-zero failure probability)
  - Hash public-key into shared key: KEM becomes contributory
  - Hash ciphertext into shared key: more robust (?)

Design space 8: CCA transforms

- General Fujisaki-Okamoto principle is the same for most KEMs (exception: NTRU)
- Tweaks to FO transform:
    - Hash public-key into coins: multitarget protection (for non-zero failure probability)
    - Hash public-key into shared key: KEM becomes contributory
    - Hash ciphertext into shared key: more robust (?)
- How to handle rejection?
    - Return special symbol (`return -1`): explicit
    - Return $\mathrm{H}(s, C)$ for secret s: implicit

Design space 8: CCA transforms

- General Fujisaki-Okamoto principle is the same for most KEMs (exception: NTRU)
- Tweaks to FO transform:
    - Hash public-key into coins: multitarget protection (for non-zero failure probability)
    - Hash public-key into shared key: KEM becomes contributory
    - Hash ciphertext into shared key: more robust (?)
- How to handle rejection?
    - Return special symbol (**`return -1`**): explicit
    - Return H(s, C) for secret s: implicit
- As of round 2, no proposal uses explicit rejection
    - Would break some security reduction
    - More robust in practice (return value alwas `0`)

Summary

- Lattice-based KEMs offer best overall performance in the PQ world
- Many tradeoffs between
  - Security (including passive vs. active)
  - Failure rate
  - Size
  - Speed
- More information about NIST PQC:
  - `https://csrc.nist.gov/projects/post-quantum-cryptography`
  - `https://pqc-wiki.fau.edu/`

Exercise: the Wookie encapsulation mechanism

Download `https://cryptojedi.org/wookie.tar.gz`
Slides at `https://cryptojedi.org/latticekems.pdf`

- CPA-secure “LPR KEM”, see slide 13
- Work in polynomial ring $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Parameters $q = 4096$, $n = 1024$
- Centered binomial noise with $k = 8$
- “Messages” have n bits $\Rightarrow$ trivial encoding (see slide 14)

Exercise: the Wookie encapsulation mechanism

Download `https://cryptojedi.org/wookie.tar.gz`
Slides at `https://cryptojedi.org/latticekems.pdf`

- CPA-secure “LPR KEM”, see slide 13
- Work in polynomial ring $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Parameters $q = 4096$, $n = 1024$
- Centered binomial noise with $k = 8$
- “Messages” have n bits $\Rightarrow$ trivial encoding (see slide 14)

1. Implement arithmetic in $\mathcal{R}_q$ (file `poly.c`)
2. Implement the Wookie KEM (file `kem.c`)

Exercise: the Wookie encapsulation mechanism

Download `https://cryptojedi.org/wookie.tar.gz`
Slides at `https://cryptojedi.org/latticekems.pdf`

- CPA-secure “LPR KEM”, see slide 13
- Work in polynomial ring $\mathcal{R}_q = \mathbb{Z}_q[X]/(X^n + 1)$
- Parameters $q = 4096$, $n = 1024$
- Centered binomial noise with $k = 8$
- “Messages” have n bits $\Rightarrow$ trivial encoding (see slide 14)

1. Implement arithmetic in $\mathcal{R}_q$ (file `poly.c`)
2. Implement the Wookie KEM (file `kem.c`)

- `make` builds various unit tests in `test/` subdirectory
- Running `test.sh` in `test/` subdirectory runs all tests

Centered binomial noise with $k = 8$

- Let HW(b) be the Hamming weight of a byte b

Centered binomial noise with $k = 8$

- Let $\mathrm{HW}(b)$ be the Hamming weight of a byte b
- To sample one coefficient $p[i]$ of a polynomial in $\mathcal{R}_q$:
  - Sample two uniformly random bytes a and b
  - Set $p[i] = \mathrm{HW}(a) - \mathrm{HW}(b)$

Centered binomial noise with $k = 8$

- Let HW(b) be the Hamming weight of a byte b
- To sample one coefficient $p[i]$ of a polynomial in $\mathcal{R}_q$:
    - Sample two uniformly random bytes a and b
    - Set $p[i] = \mathrm{HW}(a) - \mathrm{HW}(b)$
- Resulting coefficient will be in $\{-8, ..., 8\}$
- Sampling a polynomial needs $2n = 2048$ uniformly random bytes

Some remarks

- Software skeleton assumes Linux system
- Need basic build tools (`make`, `gcc`, . . .) installed:

```
apt install build-essential
```

- Some unit tests and `test.sh` script assume Sage to be installed

```
apt install sagemath
```

- Can also download pre-compiled binaries of Sage:

  https://doc.sagemath.org/html/en/installation/binary.html