MAX PLANCK INSTITUTE
FOR SECURITY AND PRIVACY

An Introduction to hash-based signatures

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NIST PQC candidates

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So many NIST candidates and one thing they all have in common. . . they all need a hash function.

What can we do with ***just*** a hash function?

Hash-based signatures

- Hash functions map long strings to fixed-length strings
- Standard properties required from a cryptographic hash function:
  - **Collision resistance:** Hard two find two inputs that produce the same output
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  - **2nd preimage resistance:** Given input and output, it's hard to find a second input, producing the same output
- Collision resistance is stronger assumption than (2nd) preimage resistance
- Ideally, don't want to rely on collision resistance

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Key generation

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  - Assume oracle $\mathcal{A}$ that computes forgery, given public key pk
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- Problem: y is not an output of h
- What if $\mathcal{A}$ can distinguish legit pk from random?
- Need additional property of h: **undetectability**
- From now on assume that all our hash functions are undetectable

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- Reduction only works with $1/2$ probability
- We get a **tightness loss** of $1/2$

One-time signatures for 256-bit messages

Key generation

- Generate 256-bit random values $s = (r_{0,0}, r_{0,1} \ldots, r_{255,0}, r_{255,1})$
- Compute $p = (h(r_{0,0}), h(r_{0,1}), \ldots, h(r_{255,0}), h(r_{255,1})) = (p_{0,0}, p_{0,1}, \ldots, p_{255,0}, p_{255,1})$

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  $\sigma = (\sigma_0, \ldots, \sigma_{255}) = (r_{0,b_0,\ldots,}r_{255,b_{255}})$

Verification

- Check that $h(\sigma_0) = p_{0,b_0}$
- $\ldots$
- Check that $h(\sigma_{255}) = p_{255,b_{255}}$

Security of this scheme

- Same idea as before, replace one $p_{j,b}$ in the public key by challenge y
- Fail if signing needs the preimage of y
- In forgery, attacker has to flip at least one bit in m
- Chance of $1/256$ that attacker flips the bit with the challenge
- Overall tightness loss of $1/512$

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- Chop 256 bit message into 64 chunks of 4 bits $m = (m_0, \ldots, m_{63})$
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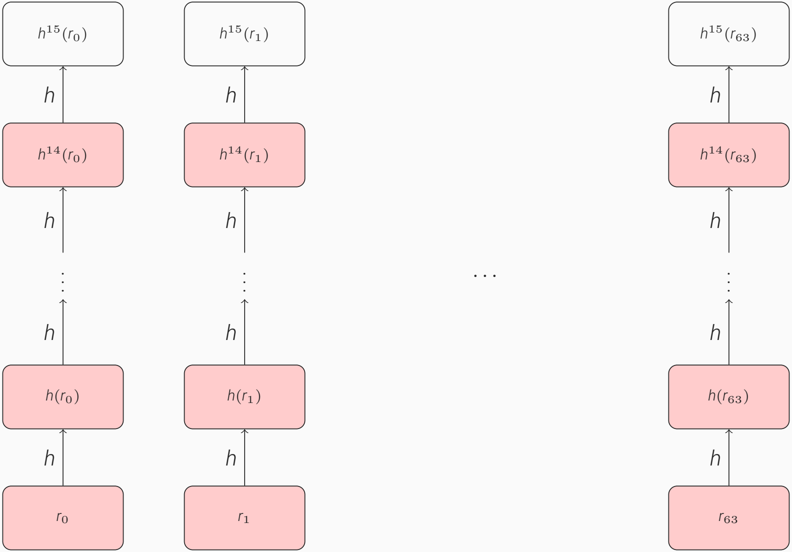
Verification

- Check that $p_0 = h^{15-m_0}(\sigma_0), \ldots, p_{63} = h^{15-m_{63}}(\sigma_{63})$

Winternitz OTS (basic idea, ctd.)



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- Once you signed, say, $m = (8, m_1, \ldots, m_{63})$, can easily forge signature on $m = (9, m_1, \ldots, m_{63})$
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- Compute $c = 960 - \sum_{i=0}^{63} m_i \in \{0, \ldots, 960\}$
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- When increasing one of the m_i's, one of the c_i's decreases
- In total obtain 67 hash chains, signatures have 2144 bytes

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    - However, $w = 256$ makes signing and verification $\approx 8\times$ slower
- Verification recovers (and compares) the full public key
- Can publish $h(\text{pk})$ instead of pk

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- Replace $h(r)$ by $h(r \oplus b)$ for "bitmask" b
- Include bitmasks in public key
- Reduction can now *choose* inputs to hash function

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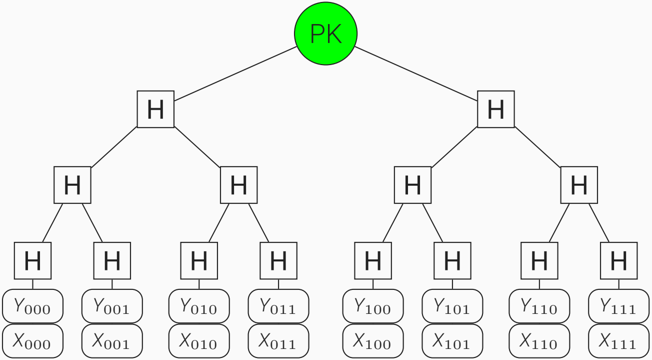
- Make deterministic: $r \leftarrow \mathrm{PRF}(s, m)$ for secret s
- Signature scheme is now **collision resilient**

Merkle Trees



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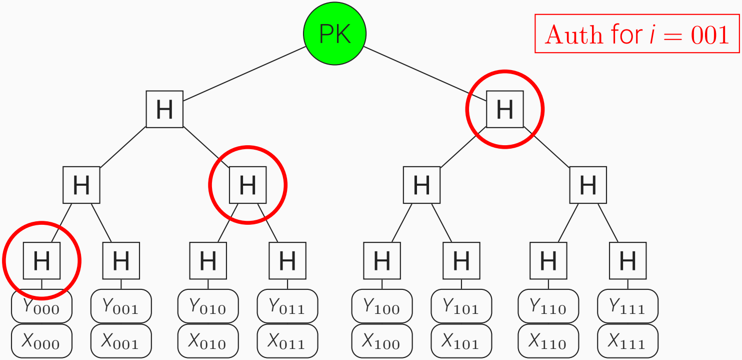

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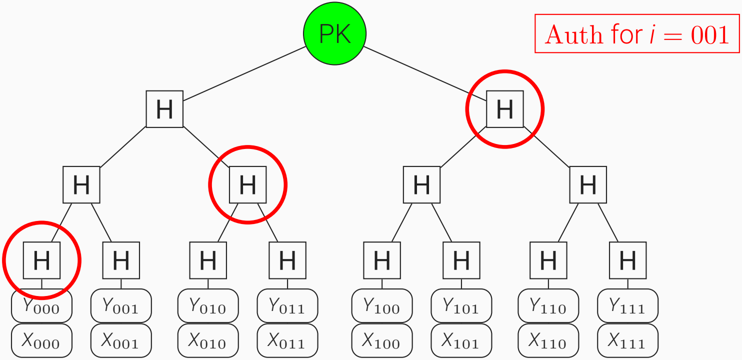

- Use OTS keys sequentially
- $\mathrm{SIG} = (i, \mathrm{sign}(M, X_i), Y_i, \mathrm{Auth})$
- Signer needs to remember current *index* ($\Rightarrow$ stateful scheme)

Merkle security

- Informally:
  - requires **EUF-CMA-secure** OTS
  - requires collision-resistant hash in the tree
- Can apply bitmask trick to get rid of collision-resistance assumption
- Merkle signatures are **stateful**

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function TREEHASH(stack, leaf node N)
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- After going through all leaves, root will be on the top of the stack
- Memory requirement: $h + 1$ hashes

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- Most of the time can reuse most nodes
- Signing speed now depends largely on index
- Idea: balance computations, store nodes required for future signatures
- Commonly used algorithm (again allowing tradeoffs): **BDS traversal** Buchmann, Dahmen, Schneider, 2008: *Merkle tree traversal revisited* http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.420.4170&rep=rep1&type=pdf

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- Huge problem in many contexts:
  - Backups
  - VM Snapshots
  - Load balancing
  - API is incompatible!

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- After key compromise publish index of compromised key
- Signatures with lower index remain valid

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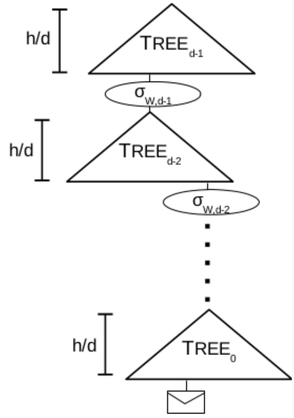
- Infeasible for very large trees
- Idea: generate all secret keys pseudo-randomly
- Use PRF on secret seed with position in the tree
- Use hierarchy of trees, **connected via one-time signatures**
- Key generation computes only the top tree
- Many more size-speed tradeoffs



SPHINCS: stateless practical hash-based signatures (2015)

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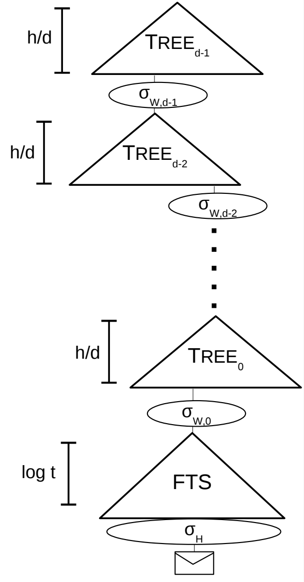
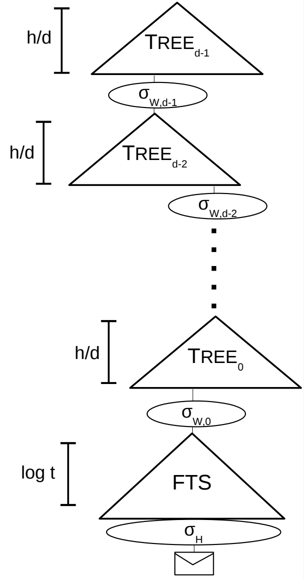
The SPHINCS approach

- Use a "hyper-tree" of total height h
- Parameter $d \geq 1$, such that $d \mid h$
- Each (Merkle) tree has height h/d
- (h/d)-ary certification tree



The SPHINCS approach

- Pick index (pseudo-)randomly
- Messages signed with *few-time* signature scheme
- Significantly reduce total tree height
- Require Pr[r-times Coll] · Pr[Forgery after r signatures] = negl(n)



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  - Each $g_i \in 0, \dots, 2^{16}$
  - Signature is $(r_{g_0}, \dots, r_{g_{31}})$
  - Signature reveals 32 out of 65536 secret-key values
  - Even after, say, 5 signatures, attacker does not know enough secret key to forge with non-negligible probability

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- $m = 512$ bit message hash (BLAKE-512)
- ChaCha12 as PRG

Cost of SPHINCS-256 signing

- Three main components:
  - PRG for HORST secret-key expansion to 2 MB
  - Hashing in WOTS and HORS public-key generation:
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- Use fast ChaCha12 permutation for π
- All building blocks (PRG, message hash, H, F) built from very similar permutations

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- Use $8\times$ parallel hashing, vectorize on high level
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SPHINCS-256 speed

- Signing: < 52 Mio. Haswell cycles (> 200 sigs/sec, 4 Core, 3GHz)
- Verification: < 1.5 Mio. Haswell cycles
- Keygen: < 3.3 Mio. Haswell cycles

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- Merge with random bitmasks into **tweakable hash function**
- NIST proposal: tweakable hash from SHA-256, SHAKE-256, or Haraka

From SPHINCS to SPHINCS$^+$, part II

- Verifiable index computation:
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    - sign digest d with FTS
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  - Verifier can check that d and i belong together
  - Attacker cannot pick d and i independently
- Additionally: Improvements to FTS (FORS)
- Use multiple smaller trees instead of one big tree
- Per signature, reveal one secret-key leaf per tree

Know more?

https://sphincs.org

Exercises

`https://cryptojedi.org/space2021-hashbased.tar.bz2`

1. Implement Lamport OTS using SHAKE-256 with 256-bit output.
   See file `lamport.c`
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More ideas (not today)

- Implement (stateful) Merkle signatures based on the Winternitz OTS from part 2, using tree height 10.
- Implement forward-secure version
- Implement configurable tradeoff between state size and speed (BDS traversal)