



Michael Naehrig

E-Vote-ID, October 2, 2025

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"style_id": "1f07eade-5cfd-a9d4-8dae-8a38ac030fe2",  
"manifest_hash": "93842E631E347297AA2E9D706C5E927AD1E15FF5262095C0BFAC",  
"code_seed": "5CAB46EB0CD84E92FA106977EDC9C5F643BCC2CF91DA414834CB92A",  
"contexts": {  
  
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    "sequence_order": 1,  
    "description_hash": "919B10E5879C72902474B87C3C294528FE555841AC26",  
    "ballot_selections": {  
  
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        "sequence_order": 1,  
        "description_hash": "9BC42537065A204F068F9488D9CB8FA11CB3DE1C",  
        "ciphertext": {  
            "pad": "94BB756344EA24B4EE5E7F5FA5283F5CA874229123593CC1A32",  
            "data": "492E124E95EB87B57F02574FEFCB8543DFAB5867990613B0CA5",  
  
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                "usage": "Prove selection's value (0 or 1)."}  
            }  
        }  
    }
```



ElectionGuard

What is it?

- a free, open-source software toolkit,
- **not** a full election system!

Election vendors can incorporate it into their existing election systems.

What it does:

- produces evidence: election record,
- enables end-to-end verifiable elections.



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- enables end-to-end verifiable elections.

Microsoft's role

- Employs researchers.
- Initiated project in 2018, publicly announced in 2019.
- Employed project manager.
- Sponsored 3rd party developers.
- Helped transition to the Election Technology Initiative in 2023.





ElectionGuard

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- a free, open-source software toolkit,
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What it does:

- produces evidence: election record,
- enables end-to-end verifiable elections.

Microsoft's current role

- Employs researchers.



Deployments



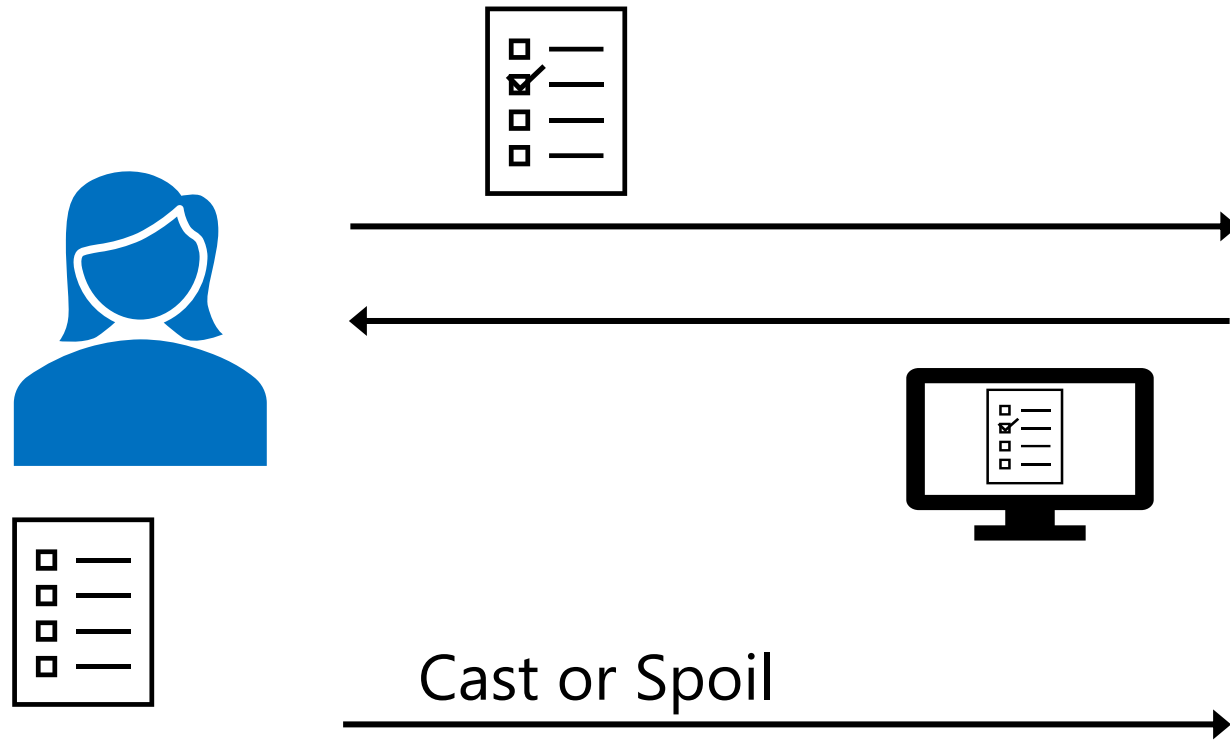
In-person Public Elections

- 2020 – Fulton, WI
with [VotingWorks](#)
- 2020 – Inyo County, CA (audit)
with [VotingWorks](#)
- 2022 – Preston, ID
with [Hart Intercivic](#)
- 2023 – College Park, MD
with [Hart Intercivic](#)

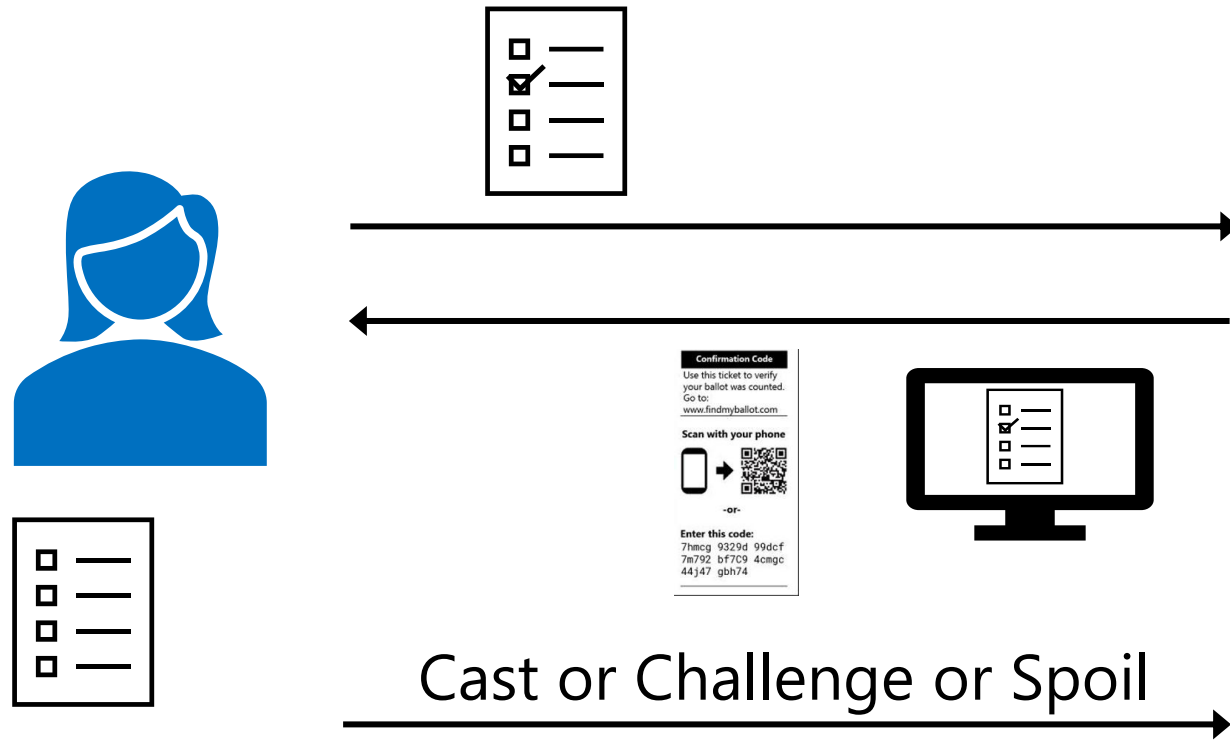
Other Elections (Remote)

- 2020 – U.S. House Democratic Caucus
with [Markup](#)
- 2023 – Neuilly-sur-Seine, FRANCE
with [Electis](#)
- 2023 – Utah absentee voters
with [Enhanced Voting](#)
- 2024 – internal election
with [Concordium](#)

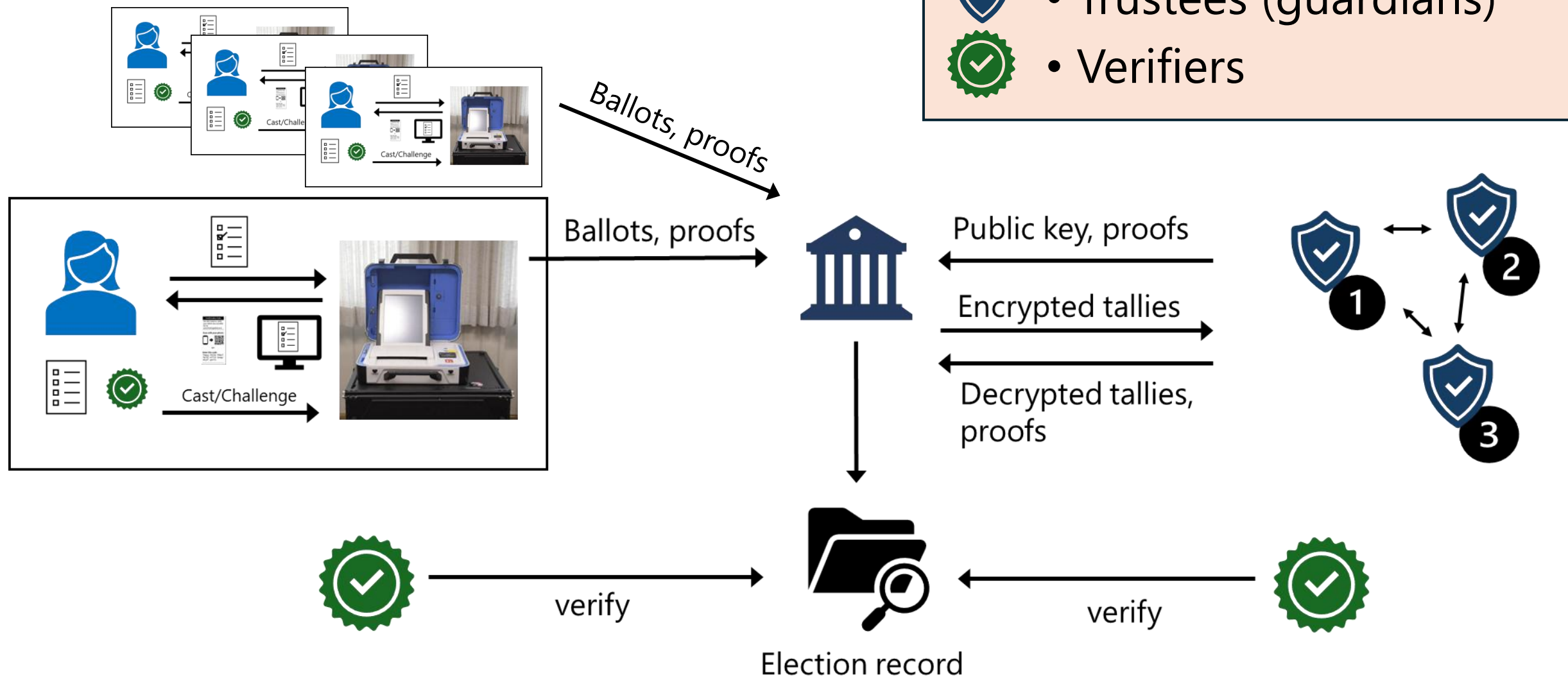
Voter experience – an example



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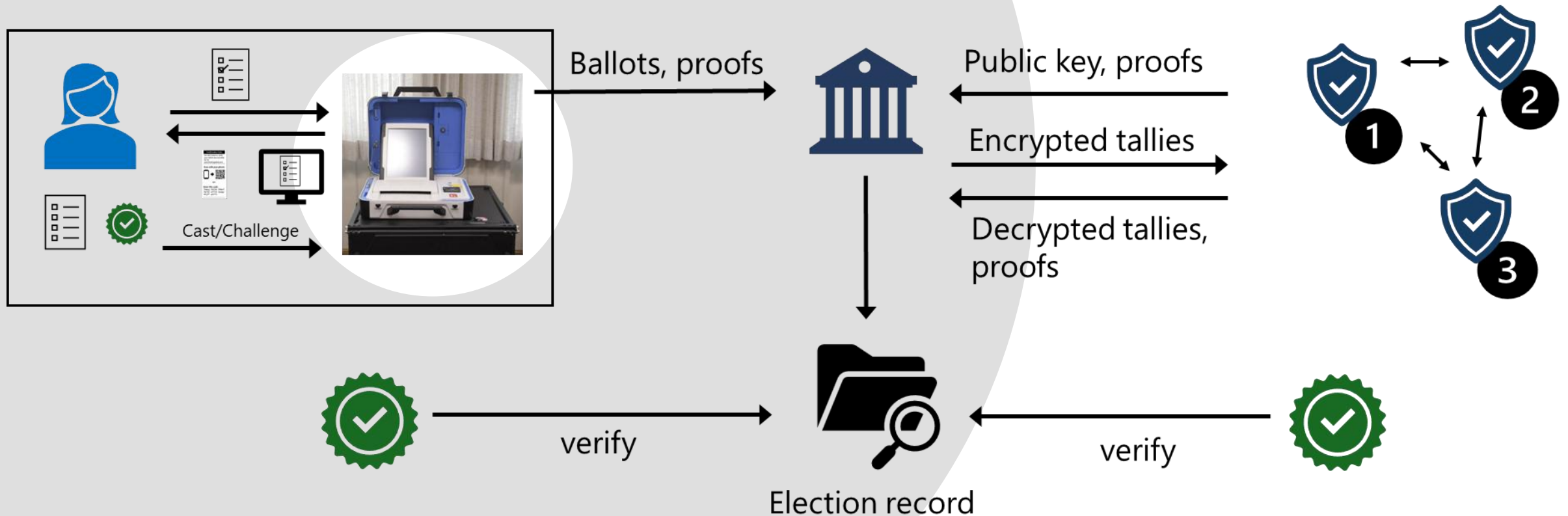


Protocol sketch



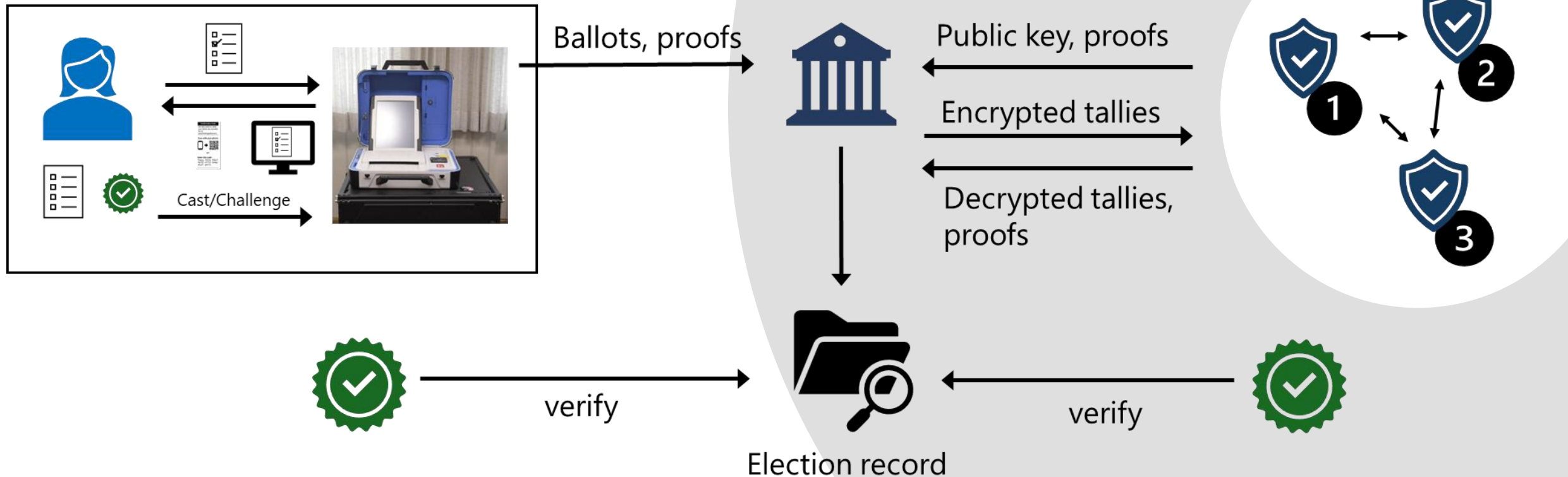
Cryptographic components

- Ballot encryption
- Proofs of ballot well-formedness
- Confirmation code



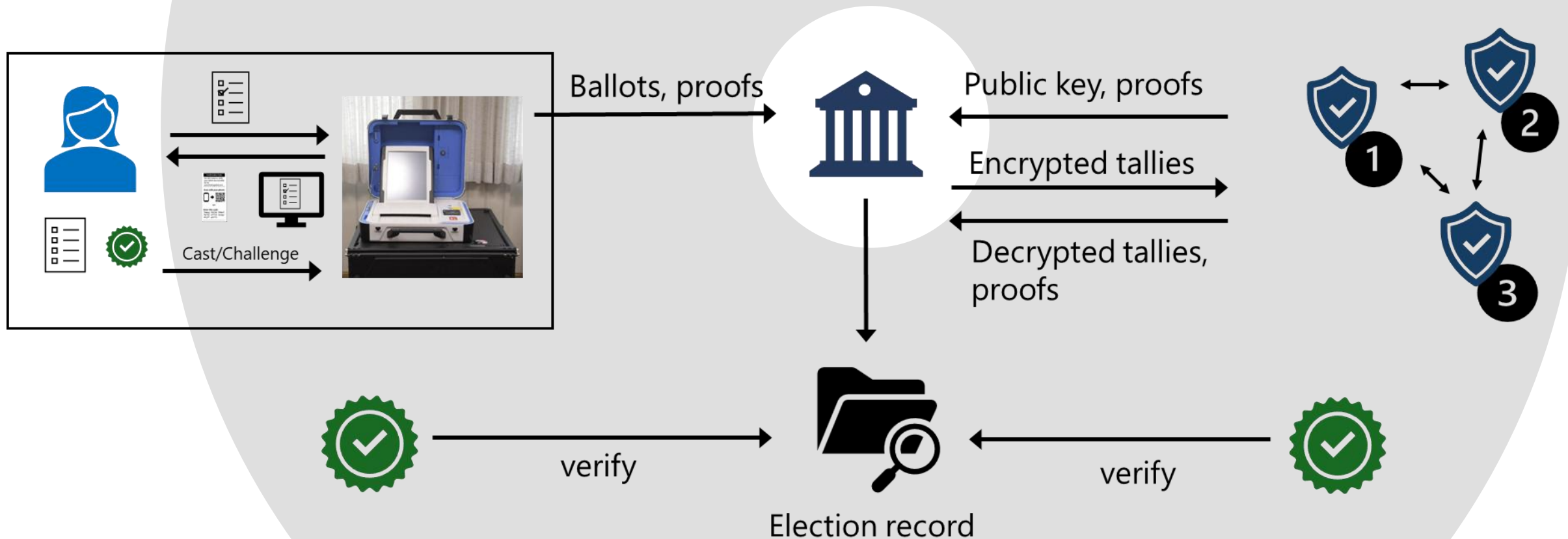
Cryptographic components

- Distributed key generation
- Verifiable decryption of tallies and challenged ballots



Cryptographic components

- Homomorphic tallying
- Facilitate decryption protocols
- Verify proofs



Cryptographic Components

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"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc1d-4204-8dae-8a38ac010fe2",
"manifest_hash": "93822f631f347227aa2b50706655227a01015ff5262079c0bf2",
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"contexts": [
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    "description_hash": "91981055879c72902474e57c3c201528f555841ac20",
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          "data": "492e174555eb05852f02574efcbb5430fa73867990613b0c0a",
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          "nonce": null,
          "proof": {
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            "proof_zero_data": "5e271d3fe3eabbb8da04253b259d55cc37ce07c",
            "proof_one_pad": "c24ad1e66aa19113f8ad333c2735c2a435f893170",
            "proof_one_data": "447585124765ac0f708f1f9430666f553559f672",
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            "proof_one_challenge": "12c53f6738a77ab7baca31ec08a4339a630",
            "challenge": "ed51316641ce79c509fd1507913c31d88049613f126ed",
            "proof_zero_response": "510807751820f903f2381038965004aed73",
            "proof_one_response": "e9cf9f60336038363a0cc05e07b186f35ef",
            "usage": "Prove selection's value (0 or 1)."
```

Cryptographic parameters

Cyclic group of order $q = 2^{256} - 189$

FF43

32B

Multiplicative group in a finite field of size p (4096 bits)

6A3D90A5EA888A04AA367E9F24CB70F7BDE7409452451A92E3D39D98CB4CFCC755572F15AE879FCE930FBBF7C08F37FCF4B42D7C58C6BDADEF
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Cryptographic parameters

Cyclic group of order $q = 2^{256} - 189$

[illegible]

32B

Multiplicative group in a finite field of size p (4096 bits)

512B

Generator g of order q

$$g^q \equiv 1 \pmod{p}$$

Operations: multiplication, exponentiation mod p

How are votes encrypted?

```
"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc4d-4204-8dae-8a38ac010fe2",
"manifest_hash": "93822f631f347297aa2b50706655227a01015ff5262095c0bf2",
"code_seed": "5ca846eb0cd84e82f4106527ed00c5f643b0c2cf910a4a4834f893a",
"contests": [
  {
    "object_id": "87e6725a-613c-43ea-a405-347c17e86315",
    "sequence_order": 1,
    "description_hash": "91981055879c72902474e57c3c201528f555841ac20",
    "ballot_selections": [
      {
        "object_id": "87e6725a-613c-43ea-a405-347c17e86315-c1fef3fe-b",
        "sequence_order": 1,
        "description_hash": "9bc425370658204f069f948829c88f411c830e10",
        "ciphertext": {
          "pad": "94b87567a4ea24b4ef5e7f8fa5283f5ca8742291235090cf1a32",
          "data": "492e174555eb05852f02874efcbb5430fa73867990613b0ca",
          "crypto_hash": "b5ca27a01750f37b671b77f4fd157358fe94f8e5ef1c6",
          "is_placeholder_selection": false,
          "nonce": null,
          "proof": {
            "proof_zero_pad": "98242316789f793f7998bdf6945b5ac4831ab951",
            "proof_zero_data": "5e271d3fe3eabbb8da64253b259d55cc37ce07c",
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            "proof_one_data": "447585124765ac0f708f1fe430666f553559f672",
            "proof_zero_challenge": "d58c472f112761704613431b8597f8361f",
            "proof_one_challenge": "12c53f6798a77ab7baca31ef08a4339a630",
            "challenge": "ed51316641ce79c509fd1507913c31d88049613f126ed",
            "proof_zero_response": "510807751820f903f2381038965094aed73",
            "proof_one_response": "e9cf9f60336838363af0c65e07b186f35ef",
            "usage": "Prove selection's value (0 or 1)."
          }
        }
      }
    ]
  }
]
```

Exponential ElGamal public key encryption

Public key Secret key

$$K = g^s$$

Encryption

$$E(m, r) = (g^r, g^m \cdot K^r)$$

Decryption with secret key

$$(g^r)^s = K^r$$
$$g^m = (g^m \cdot K^r) / (g^r)^s$$
$$m = \text{DL}_g(g^m)$$

Homomorphism

$$(g^{r_1} \cdot g^{r_2}, (g^{m_1} \cdot K^{r_1}) \cdot (g^{m_2} \cdot K^{r_2})) = (g^{r_1+r_2}, g^{m_1+m_2} \cdot K^{r_1+r_2})$$

Ciphertext

1024B



Exponential ElGamal public key encryption

Public key $\searrow$ Secret key $\swarrow$

$$K = g^s$$

Encryption

$$E(m, r) = (g^r, K^m \cdot K^r)$$

Decryption with secret key

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Ciphertext

1024B



Exponential ElGamal public key encryption

Public key $\searrow$ Secret key $\swarrow$

$$K = g^s$$

Encryption

$$E(m, r) = (g^r, K^{m+r})$$

Decryption with secret key

$$(g^r)^s = K^r$$
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Homomorphism

$$(g^{r_1} \cdot g^{r_2}, (K^{m_1+r_1}) \cdot (K^{m_2+r_2})) = (g^{r_1+r_2}, K^{(m_1+m_2)+(r_1+r_2)})$$

Ciphertext

1024B



Exponential ElGamal public key encryption

Public key $\swarrow$ Secret key $\swarrow$

$$K = g^s$$

Encryption

$$E(m, r) = (g^r, K^{m+r})$$

Decryption with secret key s

$$(g^r)^s = K^r$$
$$K^m = (K^{m+r}) / (g^r)^s$$
$$m = \text{DL}_K(K^m)$$

Decryption with random value r

$$K^r$$
$$K^m = (K^{m+r}) / K^r$$
$$m = \text{DL}_K(K^m)$$

Homomorphism

$$(g^{r_1} \cdot g^{r_2}, (K^{m_1+r_1}) \cdot (K^{m_2+r_2})) = (g^{r_1+r_2}, K^{(m_1+m_2)+(r_1+r_2)})$$

Ciphertext

1024B



Vote encryption

Encrypts a vote $m \in \{0,1\}$

$$E(m, r) = (\alpha, \beta) = (g^r, K^{m+r})$$

Option not selected: $E(0, r) = (\alpha, \beta) = (g^r, K^r)$

Option selected: $E(1, r) = (\alpha, \beta) = (g^r, K^{1+r}) = (g^r, K \cdot K^r)$

Homomorphic tallies

Multiply all encrypted votes per selection for all selections in all contests across all ballots

Encrypted votes for a single selection in a contest across all ballots

$$E(m_i, r_i) = (\alpha_i, \beta_i) = (g^{r_i}, K^{m_i+r_i})$$

Encrypted tally for that selection

$$(A, B) = (\prod \alpha_i, \prod \beta_i) = (g^{\sum r_i}, K^{\sum m_i + \sum r_i})$$

How to ensure a ballot is well-formed?

```
"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc1d-4204-8dae-8a38ac010fe2",
"manifest_hash": "93822f631f347297aa2b5070665e227a01015ff5262095c0bf2",
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            "proof_one_response": "e9cf9f60336038363af0c65e07b186f35ef",
            "usage": "Prove selection's value (0 or 1)."
```

Proof of well-formedness

Encrypts a vote $m \in \{0,1\}$

$$E(m, r) = (\alpha, \beta) = (g^r, K^{m+r}) \quad \underline{\underline{\hspace{10cm}}}$$

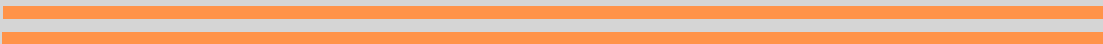
Prove that $(\alpha, \beta) = (g^r, K^r)$ is an encryption of 0 or 1 with an or-proof

	Commitment	Challenge	Response
0	$(a_0, b_0) = (g^{u_0}, K^{u_0})$	$c_0 = c - c_1$	$v_0 = u_0 - c_0 \cdot r$
		$c = H(K, \alpha, \beta, a_0, b_0, a_1, b_1)$	
1	$(a_1, b_1) = (g^{u_1}, K^{u_1 - c_1})$	c_1	$v_1 = u_1 - c_1 \cdot r$

Total size of encrypted option: (1024 + 128)B = 1152B

$$\text{Proof } \pi = (c_0, v_0, c_1, v_1) \quad \underline{\underline{\underline{\hspace{1cm}}}} \quad \mathbf{0/1} \quad 128\text{B}$$

Contest encryption

Contest			ElectionGuard
Alice		0	  0/1
Bob		0	  0/1
Carol		1	  0/1
Dave		0	  0/1

$(4 \times 1152)B = 4608B$

Selection limits

Prove that there are not more votes in a contest than allowed

Encrypted votes for all selections in contest

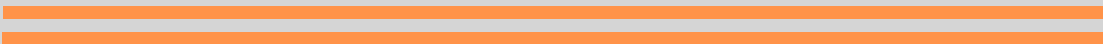
$$E(m_i, r_i) = (\alpha_i, \beta_i) = (g^{r_i}, K^{m_i+r_i})$$

Encrypted vote total in contest

$$(A, B) = (\prod \alpha_i, \prod \beta_i) = (g^{\sum r_i}, K^{\sum m_i + \sum r_i})$$

If voters must select exactly $L = 1$ option in the contest,
prove that the ciphertext encrypting the sum is an encryption of 1.

Selection limits

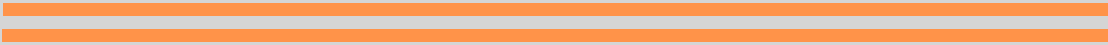
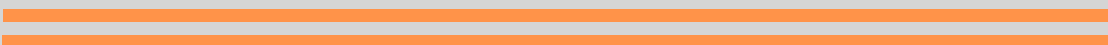
Contest			ElectionGuard
Alice		0	 ≡ 0/1
Bob		0	 ≡ 0/1
Carol	✓	1	 ≡ 0/1
Dave		0	 ≡ 0/1
Total		1	 = L=1

Range proofs

 0..1

 0..3

 0..L

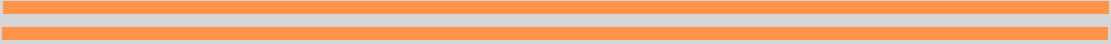
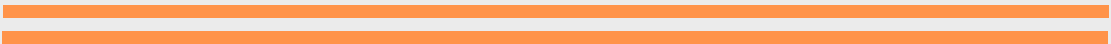
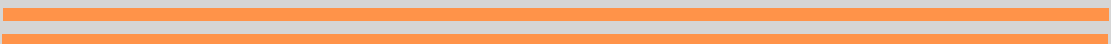
Contest			ElectionGuard
Alice		0	  0..1
Bob		0	  0..1
Carol		1	  0..1
Dave		0	  0..1
Total		1	  0..2

Range proofs

 0..1

 0..3

 0..L

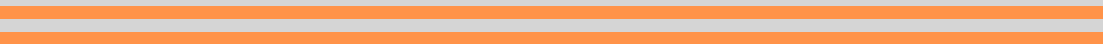
Contest			ElectionGuard	
Alice	✓	1		 0..1
Bob		0		 0..1
Carol	✓	1		 0..1
Dave		0		 0..1
Total		2		 0..2

Range proofs

 0..1

 0..3

 0..L

Contest			ElectionGuard	
Alice	✓	1		 0..3
Bob		0		 0..3
Carol	✓✓	2		 0..3
Dave		0		 0..3
Total		3		 0..3

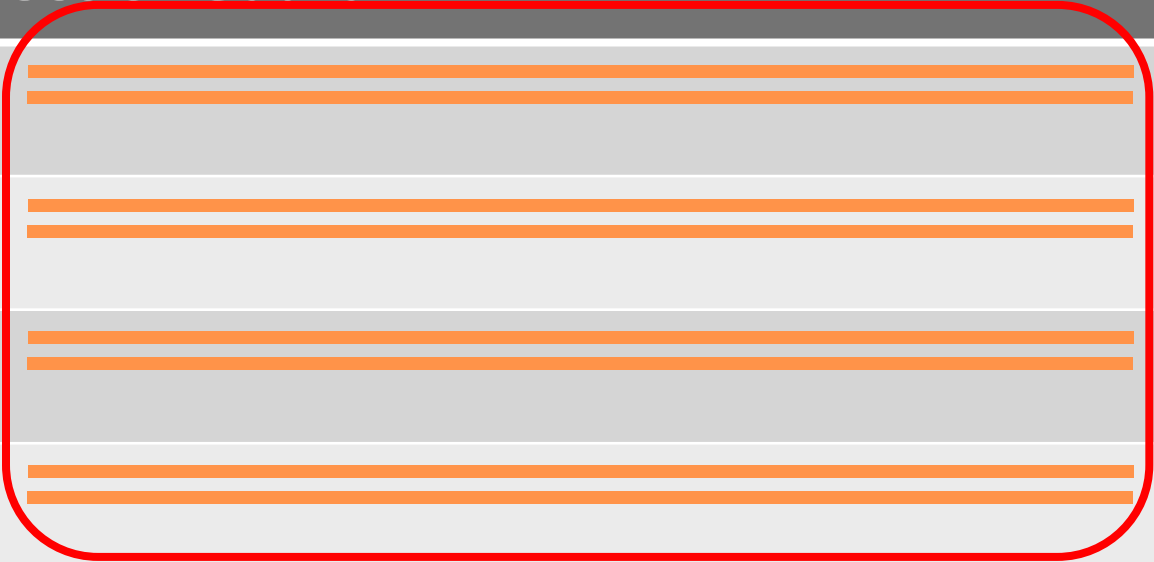
Confirmation Codes

```
"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc1d-4204-8dae-3a38ac010fe2",
"manifest_hash": "93822f631f347227aa2b50706655227a01015ff5262079c0bf2",
"code_seed": "5ca846eb0cd84e82f4106527edc0c5f643b0c2cf910a414834f8934",
"contexts": [
  {
    "object_id": "87e6745a-613c-43ea-a405-347c17e86315",
    "sequence_order": 1,
    "description_hash": "91981055879c72902474e57c3c201528f555841ac20",
    "bailot_selections": [
      {
        "object_id": "87e6745a-613c-43ea-a405-347c17e86315-c1fef3fe-b",
        "sequence_order": 1,
        "description_hash": "9bc425370658204f069f948829c08f411c030e10",
        "ciphertext": {
          "pad": "94b87567a4ea24b4ef5e7f8fa5283f5ca8742291235090cf1a33",
          "data": "492e174555eb05852f02874efcbb543df073867990613b0c0a",
          "crypto_hash": "b5c427a01750f37b671b77f4fd157358fe94f8e5ef1c6",
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          "nonce": null,
          "proof": {
            "proof_zero_pad": "98242316789f793f7998b0f6945b5ac4831ab951",
            "proof_zero_data": "5e271d3fe3eabbb8da04253b259d55cc37ce07c",
            "proof_one_pad": "c24ad1b66aa19113f8ad333c2735c2a435f893170",
            "proof_one_data": "447585124765ac0f708f1fe430666f553559f672",
            "proof_zero_challenge": "d58c472f112701704613411b8597f8361f",
            "proof_one_challenge": "12c53f6798a77ab7baca31ef08a4339a630",
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            "proof_zero_response": "510807751820f903f2381038965094aed73",
            "proof_one_response": "e9cf9f60336038363af0c65e07b186f35ef",
            "usage": "Prove selection's value (0 or 1)."
```

Confirmation codes

Voters receive a confirmation code while submitting their ballot

Hash of all vote encryptions over all contests on the ballot

Contest			ElectionGuard	
Alice		0		 0..1
Bob		0		 0..1
Carol	✓	1		 0..1
Dave		0		 0..1
Total		1		 0..1

Confirmation codes

Voters receive a confirmation code while submitting their ballot

Hash of all vote encryptions over all contests on the ballot



A red rounded rectangle containing four pairs of horizontal orange lines, representing a confirmation code.

Confirmation codes

Voters receive a confirmation code while submitting their ballot

Hash of all vote encryptions over all contests on the ballot

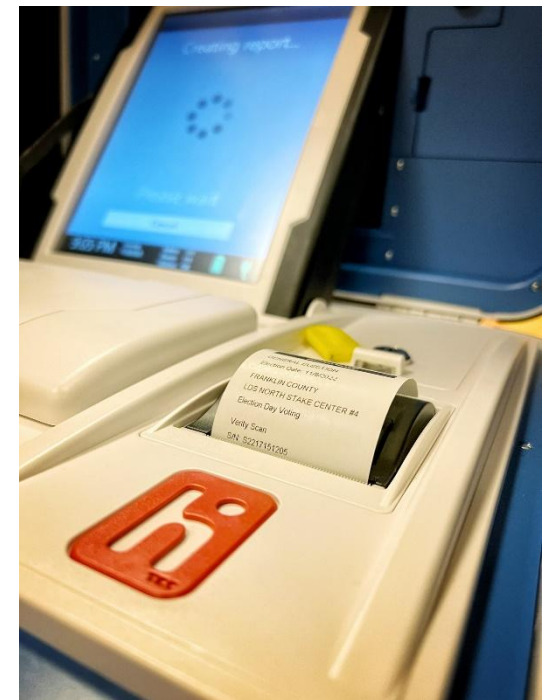


Confirmation codes

Voters receive a confirmation code while submitting their ballot

Hash of all vote encryptions over all contests on the ballot

$H(\text{[Redacted] = [Redacted]})$

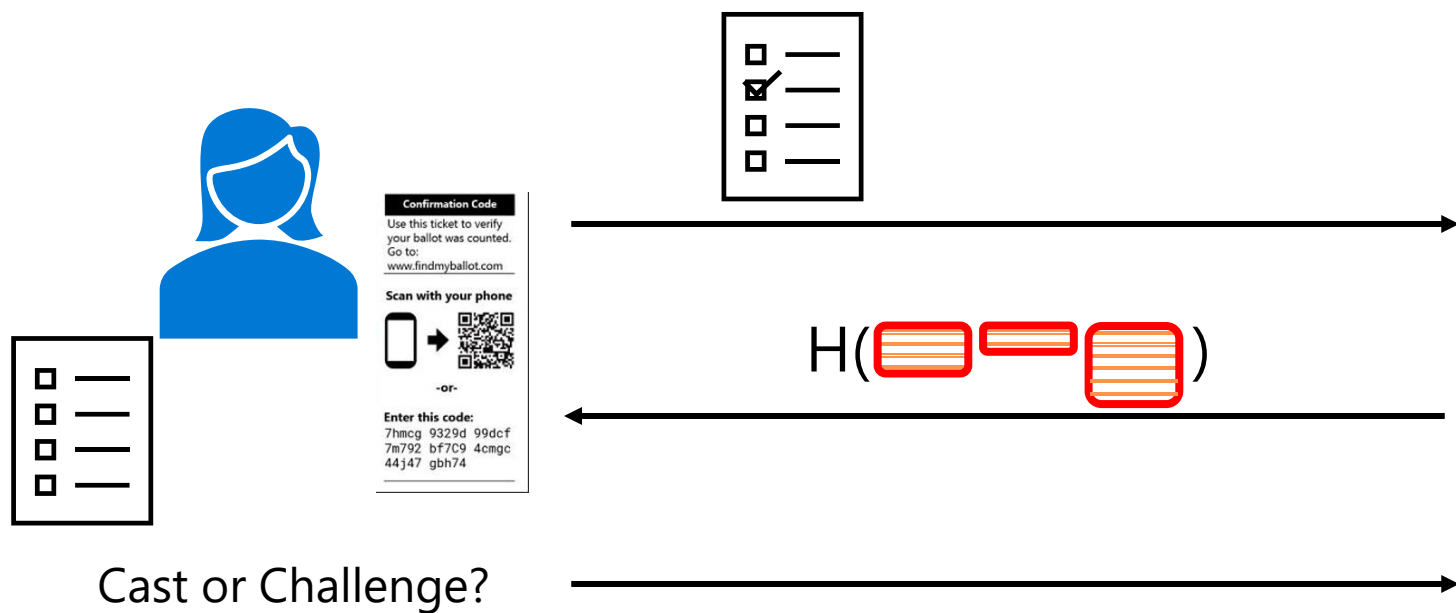


How can voters verify that their votes have been correctly encrypted?

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"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc4d-4704-8dae-8a38ac010fe2",
"manifest_hash": "93822f631f347297aa2b50706655227a01015ff5262095c0bf2",
"code_seed": "5ca846eb0cd84e82f41065977ed00c5f645bcc2cf910a4a4834f893a",
"contests": [
  {
    "object_id": "87e6725a-613c-43ea-a405-347c17e86315",
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    "description_hash": "91981055879c72902474e57c3c201528f555841ac20",
    "ballot_selections": [
      {
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        "sequence_order": 1,
        "description_hash": "9bc425370658204f069f948829c88f411c830e10",
        "ciphertext": {
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          "nonce": null,
          "proof": {
            "proof_zero_pad": "98242316789f793f7998bdf6945b5ac4831ab951",
            "proof_zero_data": "5e271d3fe3eabbb8da64253b259d55cc37ce07c",
            "proof_one_pad": "c24ad1e66aa19113f8ad33c2773ec2a435f893170",
            "proof_one_data": "447585124765ac0f708f1fe430666f553559f672",
            "proof_zero_challenge": "d58c472f112761704613411b8597f8361f",
            "proof_one_challenge": "12c53f6798a77ab7baca31ef08a4339a630",
            "challenge": "ed51316641ce79c509fd1507913c31d88049613f126ed",
            "proof_zero_response": "510807751820f903f2381038965094aed73",
            "proof_one_response": "e9cf9f60336838363af0c65e07b186f35ef",
            "usage": "Prove selection's value (0 or 1)."
```

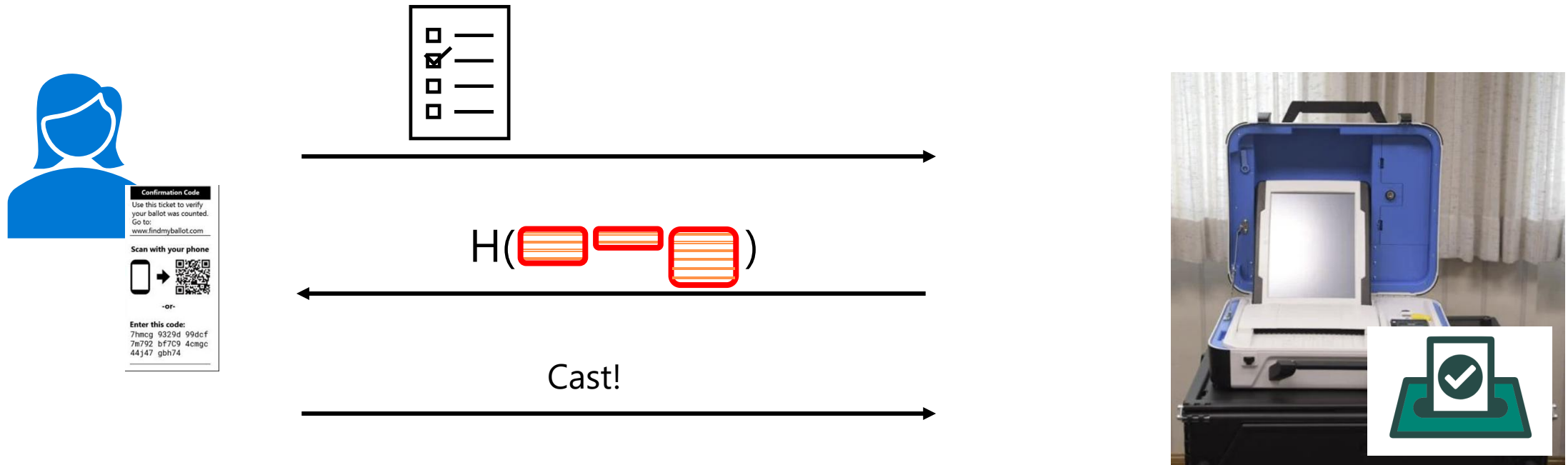
The Benaloh challenge

Challenge the encryption device in an unpredictable way.



The Benaloh challenge

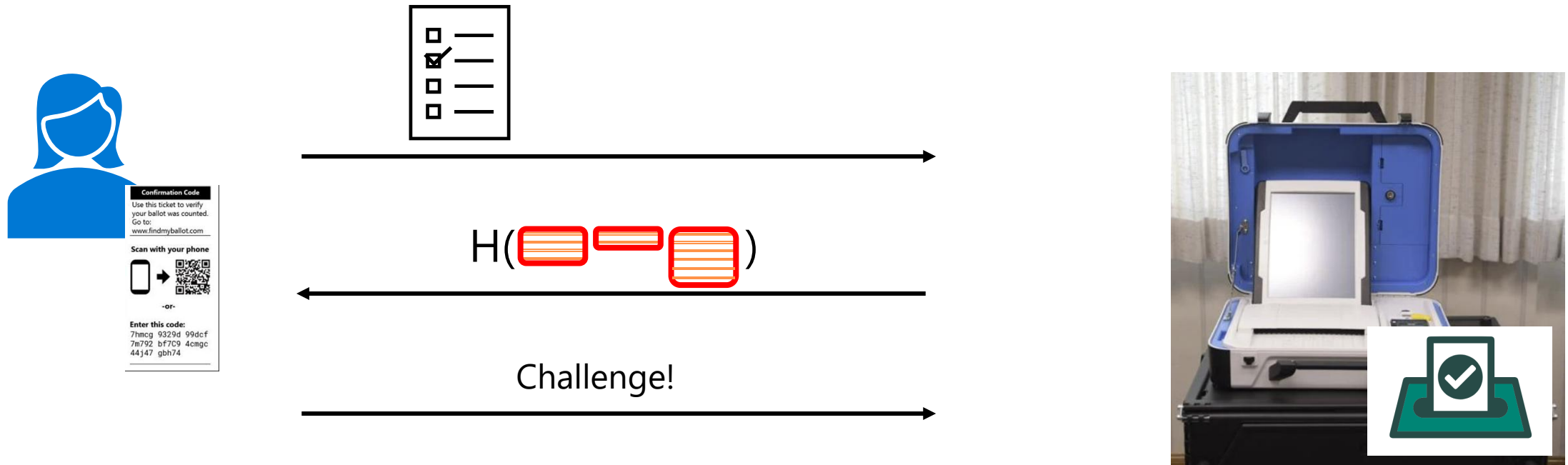
Challenge the encryption device in an unpredictable way.



If the voter decides to cast, the ballot is cast and the voting device records the encrypted ballot.

The Benaloh challenge

Challenge the encryption device in an unpredictable way.



If the voter decides to challenge, the ballot is opened and can be checked by the voter.

Who can decrypt tallies?

```
"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc4d-4204-8dae-8a38ac010fe2",
"manifest_hash": "93822f631f347297aa2b50706655227a01015ff5262095c0bf2",
"code_seed": "5ca846eb0cd84e82f41065977ed00c5f645bcc2cf910a4a4834f893a",
"contests": [
  {
    "object_id": "87e6725a-613c-43ea-a405-347c17e86315",
    "sequence_order": 1,
    "description_hash": "91981055879c72902474e57c3c201528f555841ac20",
    "ballot_selections": [
      {
        "object_id": "87e6745a-613c-43ea-a400-347c17e86315-c1fef3fe-b",
        "sequence_order": 1,
        "description_hash": "9bc425370658204f069f948829c88f411c830e1c",
        "ciphertext": {
          "pad": "94b87567a4ea24b4ef5e7f8fa5283f5ca874279123509cfa33",
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          "nonce": null,
          "proof": {
            "proof_zero_pad": "98242316789f793f7998bdf6945b5ac4831ab951",
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            "proof_one_pad": "c24ad1b66aa19113f8ad333c2735c2aa35f893170",
            "proof_one_data": "447585124765ac0f708f1fe430666f553559f672",
            "proof_zero_challenge": "d58c472f112761704613431b8597f8361f",
            "proof_one_challenge": "12c53f6798a77ab7baca31ef08a4339a630",
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            "proof_zero_response": "510807751820f903f2381038965094aed73",
            "proof_one_response": "e9cf9f60336838363a0cc65e07b186f35ef",
            "usage": "Prove selection's value (0 or 1)."
```

Election trustees and secret sharing

Election public key corresponds to a secret election key that is shared between n trustees (or guardians).

$$K_1 = g^{s_1}$$



Election public key

$$\begin{aligned} K &= K_1 \cdot K_2 \cdot K_3 \\ &= g^{s_1 + s_2 + s_3} \end{aligned}$$



$$K_3 = g^{s_3}$$



$$K_2 = g^{s_2}$$

Election secret key

$$(s = s_1 + s_2 + s_3)$$

Each guardian secret key s_i is shared via Shamir secret sharing to allow threshold decryption with k available guardians.

Verifiable decryption

Trustees jointly decrypt the tallies

Encrypted selection tally

$$(A, B) = (g^R, K^{\sum m_i + R})$$



$$M_1 = A^{s_1}$$



$$M_2 = A^{s_2}$$

$$\bar{M} = M_1 \cdot M_2 \cdot M_3 = A^S$$

$$M = B / \bar{M} = K^{\sum m_i}$$

$$\sum m_i = \text{DL}_K(M)$$

Solve a small discrete logarithm problem



$$M_3 = A^{s_3}$$

Verifiable decryption

Trustees jointly decrypt the tallies and prove correct decryption

Encrypted selection tally

$$(A, B) = (g^R, K^{\sum m_i + R})$$

Commitments



$$M_1 = A^{s_1}$$

$$(a_1, b_1) = (g^{u_1}, A^{u_1})$$

$$a = a_1 \cdot a_2 \cdot a_3 = g^{u_1 + u_2 + u_3}$$



$$M_2 = A^{s_2}$$

$$(a_2, b_2) = (g^{u_2}, A^{u_2})$$

$$b = b_1 \cdot b_2 \cdot b_3 = A^{u_1 + u_2 + u_3}$$



$$M_3 = A^{s_3}$$

$$(a_2, b_2) = (g^{u_2}, A^{u_2})$$

Verifiable decryption

Trustees jointly decrypt the tallies and prove correct decryption

Encrypted selection tally

$$(A, B) = (g^R, K^{\sum m_i + R})$$

Commitments



$$M_1 = A^{s_1}$$

$$(a_1, b_1) = (g^{u_1}, A^{u_1})$$

$$a = a_1 \cdot a_2 \cdot a_3 = g^{u_1 + u_2 + u_3}$$



$$M_2 = A^{s_2}$$

$$(a_2, b_2) = (g^{u_2}, A^{u_2})$$

$$b = b_1 \cdot b_2 \cdot b_3 = A^{u_1 + u_2 + u_3}$$



$$M_3 = A^{s_3}$$

$$(a_2, b_2) = (g^{u_2}, A^{u_2})$$

$$v = v_1 + v_2 + v_3$$

$$\pi = (c, v)$$



Proof

Public evidence

```
"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc4d-4704-8dae-3a38ac010fe2",
"manifest_hash": "93822f631f347297aa2b50706655227a01015ff5262095c0bf2",
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    "object_id": "87e6725a-613c-43ea-4065-347c17e86315",
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        "object_id": "87e6725a-613c-43ea-4065-347c17e86315-c1fef3fe-b",
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          "data": "492f174b55eb07852f02874efcbb5430fa73867990613b0c0a",
          "crypto_hash": "b5c427a01750f37b671b77f4fd157358fe94f8e5ef1c6",
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            "proof_zero_data": "5e271d3fe3eabbb8da04253b259d55cc37ce07c",
            "proof_one_pad": "c24ad1b66aa19113f8ad330c2735c2aa35f893170",
            "proof_one_data": "447585124765ac0f708f1fe430666f653459f672",
            "proof_zero_challenge": "d58c472f112761704613411b8597f8361f",
            "proof_one_challenge": "12c53f6798a77ab7baca31ef08a4339a630",
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            "proof_zero_response": "510807751820f903f2381038965094aed73",
            "proof_one_response": "e9cf9f60336838363a0cc65e07b186f35ef",
            "usage": "Prove selection's value (0 or 1)."
```

The election record



The **public record** that provides the **evidence** to verify the election.

- Election manifest
- Encrypted submitted ballots
- Challenged ballots
- Encrypted and decrypted tallies
- Cryptographic proofs that the above are well-formed and correct

The election record is large:

Example ballot: 17 contests,
47 selectable options total

Ciphertexts + proofs:

===== ≡ 0/1

$47 * 1152B + 17 * 1152B$

$= 73728B > 70KB$

~10000 ballots: $> 700MB$

What have we learned from pilots?

Challenges:



- Performance, interplay between parameters and devices

H

- Specifying how data is hashed is important!



- Size of the election record



- Challenge process in real time?



- Trustee process is complicated and requires interaction.
For it to be meaningful, trustees must understand their role

What's next?

```
"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc4d-4704-8dae-3a38ac010fe2",
"manifest_hash": "93822f631f347297aa2b50706655227a01015ff5262095c0bf2",
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    "bailot_selections": [
      {
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        "description_hash": "9bc425370658204f069f948824c88f411c830e10",
        "ciphertext": {
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          "data": "492e174555eb05852f02874efcbb5430fa73867990613b0ca",
          "crypto_hash": "b5ca27a01750f37b671b77f4fd157358fe94f8e5ef1c6",
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          "nonce": null,
          "proof": {
            "proof_zero_pad": "98242316789f793f7998bdf6945b5ac4831ab951",
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            "proof_one_pad": "c24ad1b66aa19113f8ad333c2735c2a435f893170",
            "proof_one_data": "447585124765ac01708f1fe430666f653459f672",
            "proof_zero_challenge": "d58c472f112761704613431b8597f8361f",
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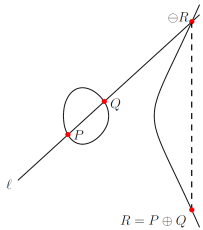
Possible changes and extensions



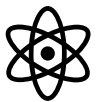
- Add functionality for verifiable mixnets.



- Replace human trustees with secure hardware?

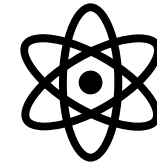


- Use elliptic curve group instead of finite field group.



- Make EG post-quantum.

Post-quantum cryptography



- A large-scale quantum computer would be able to compute discrete logarithms in polynomial time
- Breaks ElGamal encryption and thus privacy of the votes
- Use post-quantum primitives, e.g. lattice-based cryptography

Algorithms for Quantum Computation: Discrete Logarithms and Factoring

Peter W. Shor
AT&T Bell Labs
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600 Mountain Ave.
Murray Hill, NJ 07974, USA

Abstract

A computer is generally considered to be a universal computational device; i.e., it is believed able to simulate any physical computational device with a cost in computation time of at most a polynomial factor. It is not clear whether this is still true when quantum mechanics is taken into consideration. Several researchers, starting with David Deutsch, have developed models for quantum mechanical computers and have investigated their computational properties. This paper gives Las Vegas algorithms for finding discrete logarithms and factoring integers on a quantum computer that take a number of steps which is polynomial in the input size, e.g., the number of digits of the integer to be factored. These two problems are generally considered hard on a classical computer and have been used as the basis of several proposed cryptosystems. (We thus give the first examples of quantum cryptanalysis.)

[1, 2]. Although he did not ask whether quantum mechanics conferred extra power to computation, he did show that a Turing machine could be simulated by the reversible unitary evolution of a quantum process, which is a necessary prerequisite for quantum computation. Deutsch [9, 10] was the first to give an explicit model of quantum computation. He defined both quantum Turing machines and quantum circuits and investigated some of their properties.

The next part of this paper discusses how quantum computation relates to classical complexity classes. We will thus first give a brief intuitive discussion of complexity classes for those readers who do not have this background. There are generally two resources which limit the ability of computers to solve large problems: time and space (i.e., memory). The field of analysis of algorithms considers the asymptotic demands that algorithms make for these resources as a function of the problem size. Theoretical computer scientists generally classify algorithms as efficient when the number of steps of the algorithms grows as a polynomial in the size of the input. The class of problems which can be solved by efficient algorithms is known as P. This classification has several nice properties. For one thing, it does a reasonable job of reflecting the performance of algorithms in practice (although an algorithm whose running time is the tenth power of the input size,

1 Introduction

Since the discovery of quantum mechanics, people have found the behavior of the laws of probability in quantum mechanics counterintuitive. Because of this behavior,

Elliptic Curves

Replace finite field group with elliptic curve group

$$E/\mathbb{F}_p: y^2 = x^3 - 3x + b$$

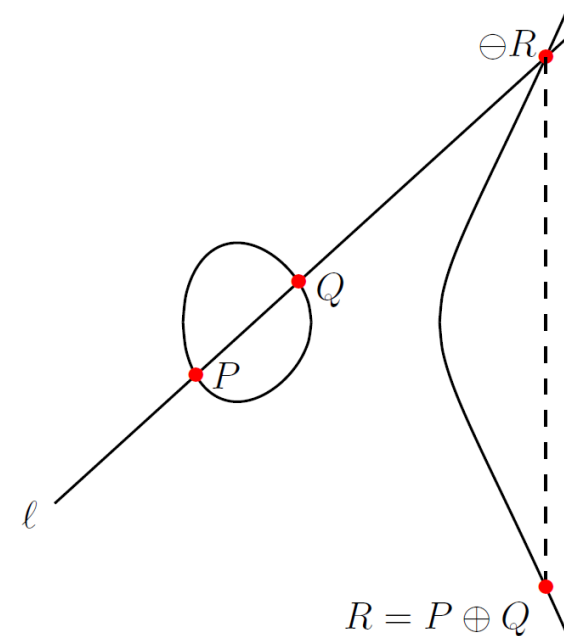
P-384

$$p = 2^{384} - 2^{128} - 2^{96} + 2^{32} - 1$$

$p = 39402006196394479212279040100143613805079739270465446667948293404245721771496870329047266088258938001861606973112319$

3940200619639447921227904010014361380507973927046544666794
8293404245721771496870329047266088258938001861606973112319

3940200619639447921227904010014361380507973927046544666794
8293404245721771496870329047266088258938001861606973112319



Elliptic Curves

Replace finite field group with elliptic curve group

$$E/\mathbb{F}_p: y^2 = x^3 - 3x + b$$

P-384

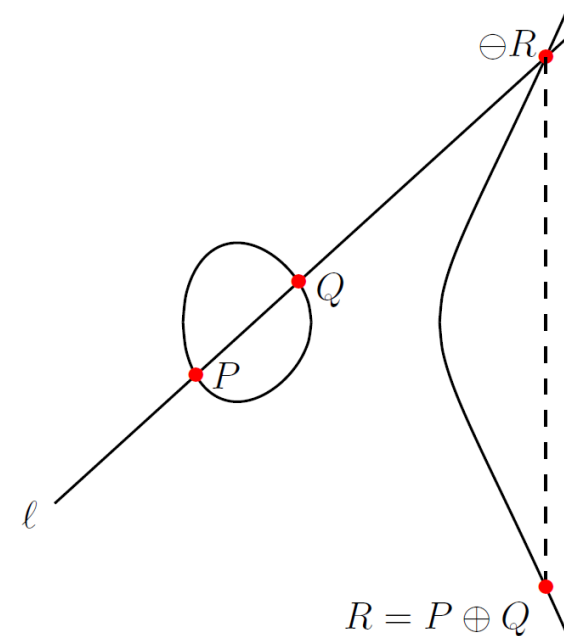
$$p = 2^{384} - 2^{128} - 2^{96} + 2^{32} - 1$$

$p = 39402006196394479212279040100143613805079739270465446667948293404245721771496870329047266088258938001861606973112319$

48B

$$G = (26247035095799689268623156744566981891852923491109213387815615900925518854738050089022388053975719786650872476732087, 8325710961489029985546751289520108179287853048861315594709205902480503199884419224438643760392947333078086511627871)$$

$$G = (\text{---}, \text{---}) \rightarrow G = \text{---}$$



Elliptic Curves

- Group elements are smaller for similar security
- Ciphertext can be represented by 2 elements

$$2 * 48B = 96B$$

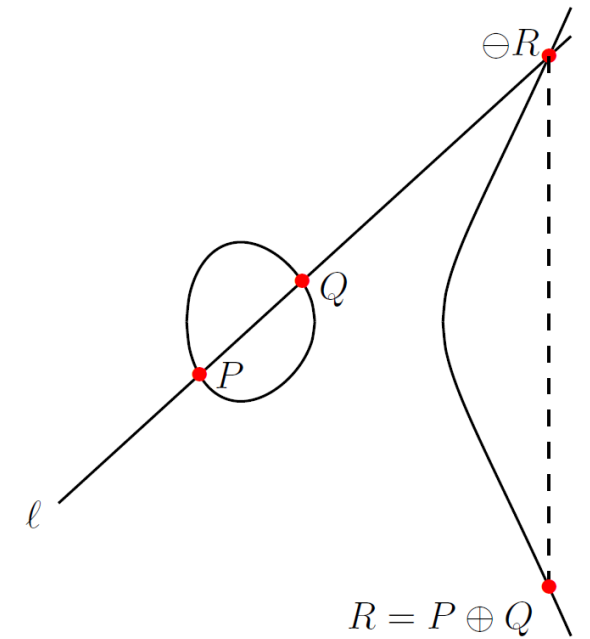


- Proofs become slightly larger

$$4 * 48B = 192B$$



0/1



  0/1

  0/1

- Example ballot from earlier:

$$64 * 192B = 12288B > 12KB$$

$$10000 \text{ ballots} > 120MB$$

New directions

```
"object_id": "1402450000000000000051",
"style_id": "1f02eade-bc4d-4704-8dae-8a38ac010fe2",
"manifest_hash": "93822f631f347297aa2b50706655227a01015ff5262095c0bf2",
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      {
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        "sequence_order": 1,
        "description_hash": "9bc425370658204f069f948824c88f411c830e10",
        "ciphertext": {
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          "data": "492e174b55eb05852f02874efcbb5430fa73867990613b0c0a",
          "crypto_hash": "b5ca7a01750f37b671b77f4fd157358fe94f8e5ef1c6",
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          "nonce": null,
          "proof": {
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            "proof_zero_data": "5e271d3fe3eabbb8da04253b259d55cc37ce07c",
            "proof_one_pad": "c24ad1b66aa19113f8ad330c2735c2aa35f893170",
            "proof_one_data": "447585124765ac01708f1fe430666f653459f672",
            "proof_zero_challenge": "d58c472f112761704613431b8597f8361f",
            "proof_one_challenge": "12c53f6798a77ab7baca31ef08a4339a630",
            "challenge": "ed51316641ce79c509fd1507913c31d88049613f126ed",
            "proof_zero_response": "510807751820f903f2381038965094aed73",
            "proof_one_response": "e9cf9f60336838363a0cc65e07b186f35ef",
            "usage": "Prove selection's value (0 or 1)."
```

Commitments

Replace encryption with commitments

Encryption

$$E(m, r) = (g^r, K^{m+r})$$

$$K = g^s$$

Commitment

$$C(m, r) = g^r \cdot g_1^m$$

- Generators g and g_1 are generated publicly without knowledge of discrete logarithm between them.
- There are no keys anymore!
- Statistically hiding, computationally binding. Everlasting privacy!

Commitments

Replace encryption with commitments

Encryption

$$E(m, r) = (g^r, K^{m+r})$$

Encryption homomorphism

$$\begin{aligned} & (g^{r_1} \cdot g^{r_2}, (K^{m_1+r_1}) \cdot (K^{m_2+r_2})) \\ &= (g^{r_1+r_2}, K^{(m_1+m_2)+(r_1+r_2)}) \end{aligned}$$

Commitment

$$C(m, r) = g^r \cdot g_1^m$$

Commitment homomorphism

$$\begin{aligned} & (g^{r_1} \cdot g_1^{m_1}) \cdot (g^{r_2} \cdot g_1^{m_2}) \\ &= (g^{r_1+r_2} \cdot g_1^{m_1+m_2}) \end{aligned}$$

Commitments

Replace encryption with commitments

Encryption

$$E(m, r) = (g^r, K^{m+r})$$

$$E(m_1, r_1) = (g^{r_1}, K^{m_1+r_1}) \underline{\underline{\hspace{1cm}}}$$

$$E(m_2, r_2) = (g^{r_2}, K^{m_2+r_2}) \underline{\underline{\hspace{1cm}}}$$

$$E(m_3, r_3) = (g^{r_3}, K^{m_3+r_3}) \underline{\underline{\hspace{1cm}}}$$

$$E(m_4, r_4) = (g^{r_4}, K^{m_4+r_4}) \underline{\underline{\hspace{1cm}}}$$

Commitment

$$C(m, r) = g^r \cdot g_1^m$$

$$C(m_1, r_1) = g^{r_1} \cdot g_1^{m_1} \underline{\hspace{1cm}}$$

$$C(m_2, r_2) = g^{r_2} \cdot g_1^{m_2} \underline{\hspace{1cm}}$$

$$C(m_3, r_3) = g^{r_3} \cdot g_1^{m_3} \underline{\hspace{1cm}}$$

$$C(m_4, r_4) = g^{r_4} \cdot g_1^{m_4} \underline{\hspace{1cm}}$$

Commitments

Replace encryption with commitments

Encryption

$$E(m, r) = (g^r, K^{m+r})$$

Contest			Encryption
Alice		0	0/1
Bob		0	0/1
Carol	✓	1	0/1
Dave		0	0/1

Commitment

$$C(m, r) = g^r \cdot g_1^m$$

Contest			Commitment
Alice		0	? 0/1
Bob		0	? 0/1
Carol	✓	1	? 0/1
Dave		0	? 0/1

Tallying committed votes

Commitments to votes

$$C(m_i, r_i) = g^{r_i} \cdot g_1^{m_i}$$

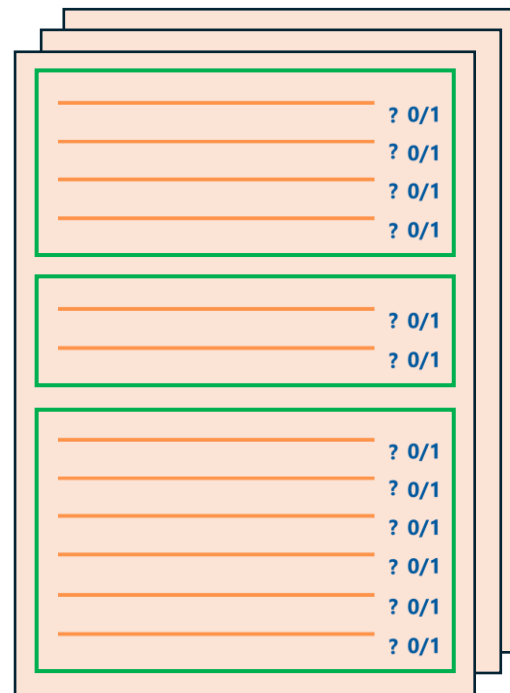
Device keeps

Sum of random values, running tally, commitments

$$\sum m_i, \sum r_i, C(m_i, r_i)$$

Commitment to the tally for that selection

$$C = \prod C(m_i, r_i) = g^{\sum r_i} \cdot g_1^{\sum m_i}$$



Contest	$\sum m_i$	$\sum r_i$
Alice	314	—
Bob	47	—
Carol	376	—
Dave	12	—

Vector commitments

$$C(\vec{m}, r) = g^r \cdot g_1^{m_1} \cdot g_2^{m_2} \cdot g_3^{m_3} \cdot g_4^{m_4} \cdot \dots \cdot g_n^{m_n}$$

- Per contest:
A single $C(\vec{m}, r) =$ _____
- Proof (with n selections):
_____ + $(2 \cdot n + 6)$ —
- Per selection: 2 —
Add 2 _____ + 6 — per contest

Example ballot:

$$47 \cdot 2 \cdot 32B + 17 \cdot (2 \cdot 512B + 6 \cdot 32B) = 23680B > 23KB$$

Elliptic curve P-384:

$$47 \cdot 2 \cdot 48B + 17 \cdot (2 \cdot 48B + 6 \cdot 48B) = 11040B > 11KB$$

Elliptic curve P-256:

$$47 \cdot 2 \cdot 32B + 17 \cdot (2 \cdot 32B + 6 \cdot 32B) = 7360B > 7KB$$

Committing the whole ballot:

$$47 \cdot 2 \cdot 32B + 17 \cdot 2 \cdot 32B + 2 \cdot 32B + 6 \cdot 32B = 4352B > 4KB$$



ElectionGuard

<https://www.electionguard.vote/>

<https://github.com/Election-Tech-Initiative/electionguard>

<https://electiontechnology.org/>

Design specification (Josh Benaloh, M. N., Olivier Pereira):

<https://www.electionguard.vote/spec/>

Academic paper (Josh Benaloh, M. N., Olivier Pereira, Dan S. Wallach):

<https://dl.acm.org/doi/10.5555/3698900.3699207>

v2.0 implementations (John Caron):

<https://github.com/JohnLCaron/egk-ec>

The MITRE verifier:

<https://electionintegrity.mitre.org/verifier/>



ElectionGuard

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